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Abstract*

This paper discusses the role of sterilized foreign exchange (FX) interventions as a monetary policy instrument for emerging market economies in response to external shocks. We develop a model for a commodity-exporting small open economy in which FX intervention is considered as a balance sheet policy induced by a financial friction in the form of an agency problem between banks and their creditors. The severity of banks' agency problem depends directly on a bank-level measure of currency mismatch. Endogenous deviations from the standard UIP condition arise at equilibrium. In this context, FX interventions moderate the response of financial and macroeconomic variables to external shocks by leaning against the wind with respect to real exchange rate pressures. Our quantitative results indicate that, conditional on external shocks, the FX intervention policy successfully reduces credit, investment, and output volatility, along with substantial welfare gains when compared to a free-floating exchange rate regime. Finally, we explore distinct generalizations of the model that eliminate the presence of endogenous UIP deviations. In those cases, FX intervention operations are considerably less effective for the aggregate equilibrium.

JEL classifications: E32, E44, E52, F31, F41
Keywords: Foreign exchange intervention, External shocks, Monetary policy, Financial dollarization, Financial frictions

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1. Introduction

Emerging market economies (EMEs) face volatile external shocks that have shaped capital flows and exchange rate dynamics since the collapse of the Bretton Woods system and more recently due to global financial integration. These external shocks have different fundamentals which can be summarized in terms of three main interrelated components: global demand, foreign interest rates, and commodity prices. For instance, some relatively recent global events that had significant implications for EMEs are: the global commodity boom originating from China's strong demand during the 2000s, the expansionary monetary policies in major advanced economies in response to the Global Financial Crisis, and the normalization of the Fed's accommodative monetary policy (also known as the Taper Tantrum). At the same time, these capital flows to EMEs affect domestic financial conditions and credit growth through the availability of foreign currency-denominated funds and exchange rate fluctuations, which in some cases have placed the financial system in a more fragile situation.

Many central banks, especially in EMEs, responded to these events by building FX reserves during capital inflow episodes. These central banks were considered to be in a good position to deal with capital reversals and effectively sold those accumulated reserves during capital outflow episodes. Specifically, EMEs have relied on sterilized FX interventions (i.e., official FX purchases or sales aimed at leaving domestic liquidity unaffected) to smooth out the impact of rapidly shifting capital flows and reduce exchange rate volatility while providing businesses and households with insurance against exchange rate risks. Moreover, foreign currency debt in EMEs has increased, leaving them more exposed to global financial flows; and therefore financial stability has become an important objective of FX interventions.[1] Additionally, the mix of policy tools used by policy makers in EMEs also includes macro-prudential measures and capital controls.[2] The effectiveness of these tools is still under debate, and more research is needed to make a better assessment of these instruments as a complement to conventional interest rate policy.

The purpose of this paper is to develop a macroeconomic model to analyze FX interventions as a monetary policy tool that takes on attributes of a financial stability instrument

[1] The existing literature have identified four main policy objectives for using FX interventions: financial stability, price stability, precautionary savings (after experiencing crisis in the 1980s and 1990s), and export competitiveness. In this paper, we focus on the first two. See Arslan and Cantú (2019), Patel and Cavallino (2019), Chamon and Magud (2019), Hendrick et al. (2019), and Chamon et al. (2019).

[2] See Céspedes et al. (2014) for a discussion of recent LATAM central banks' experiences.

as a response to external shocks. We define (sterilized) FX interventions, as a situation where the central bank buys/sells FX with the banking system in exchange for domestic currency-denominated bonds issued by the central bank, but in a way that offsets any change in the supply of domestic liquidity. In line with Chang (2019), we view FX intervention operations as a non-conventional monetary tool induced by the existence of financial frictions in the domestic banking sector. In particular, when the relevant financial friction binds, leverage constraints restrict banks' balance sheet capacity and limits to arbitrage emerge together with widening interest rate spreads. In the financially constrained equilibrium, however, FX interventions affect the equilibrium real allocation, since it relaxes or tightens the financial constraint that banks face.[3]

In our framework, FX interventions affect the economy via two mutually reinforcing effects: exchange rate stabilization and lending capacity crowding out induced by the sterilization process associated with the FX intervention policy (similar to the empirical findings of Hofmann et al., 2019).[4] We suggest, however, that the financial friction approach to FX interventions differs from unconventional monetary policy for closed economies in several aspects. The unconventional monetary policy literature emphasizes that the conventional instrument is active until the policy rate reaches the effective lower bound. Only in those cases, central banks might deploy balance sheet policies such as QE, LSAP, or credit policies. On the contrary, we consider that financial constraints are binding in EMEs even in normal times. Consequently, we argue that, for inflation targeter EMEs, FX interventions might be considered a balance sheet policy that is active in normal times, as well as during credit crunch or sudden stop episodes. Contrary to Chang (2019), we suggest that what really matters in EMEs is how tight financial constraints are and not necessarily if those constraints bind.

We build a general equilibrium model for a commodity-exporting small open economy where FX intervention operations are relevant for the equilibrium allocation. In our framework, the central bank follows a Taylor rule to set its monetary policy rate (conventional monetary policy) but also "leans against the wind" in response to exchange rate fluctuations. The model is an extension of Aoki et al. (2018), henceforth ABK, where banks face an agency problem that

[3] In addition, our model considers limited participation of households with respect to foreign currency denominated bank deposits. Both, banks and households, face limits to arbitrage between domestic and foreign currency-denominated assets/liabilities. The relevance of each friction for the effectiveness of FX intervention policy is discussed in Section 5.

[4] See Céspedes et al. (2017), Chang (2019), and Céspedes and Chang (2019) for similar frameworks that introduce FX interventions as an unconventional policy tool.

constrains their ability to obtain funds from domestic households and international financial markets. Like in Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Gertler et al. (2012), and Gertler and Karadi (2013), the agency problem introduces an endogenous leverage constraint that relates credit flows to banks' net worth and ultimately makes the balance sheet of the banking sector a critical determinant of the cost of credit faced by borrowers. In this context, unconventional monetary policies or balance sheet policies, such as FX intervention, have real effects.

Our model departs from ABK in three key aspects. First, the banking system is partially dollarized on both sides of its balance sheet and exposed to potential currency mismatches and sudden exchange rate depreciations, as is the case in many EMEs that show a high degree of vulnerability to external shocks. Therefore, credit and deposit dollarization coexist in equilibrium as endogenous variables. On one hand, we assume that intermediate good producers must borrow in advanced from banks in order to acquire capital for production but needs a combination of domestic currency and foreign currency-denominated loans to buy capital. The combination of both types of loans is achieved assuming a Cobb-Douglas technology that yields a unit measure of aggregate loan services. As a result, the asset composition of banks is given by loans in domestic and foreign currency in addition to holdings of bonds issued by the central bank for sterilization purposes. On the other hand, we assume that households are allowed to hold deposits with banks that are denominated in domestic and foreign currency. However, we introduce limits on household foreign currency-denominated deposits by assuming transaction costs as a simple way to capture incomplete arbitrage.

Second, the severity of the bank's agency problem depends directly on a measure of currency mismatch at the bank level given by the difference between dollar-denominated liabilities and assets as a fraction of total assets. However, not all assets enter symmetrically into the banks' incentive compatibility constraint that characterizes the agency problem. In particular, central bank assets are harder to divert than private loans. Third, the central bank "leans against the wind" regarding exchange rate pressures due to external shocks, but in a sterilized manner. In our setting, an FX intervention policy is a balance sheet operation that takes place when the central bank sells dollars to, or buys dollars from, the banking system in exchange for domestic currency-denominated assets. However, it does so in a way that completely offsets any change in the supply of domestic liquidity by using domestic bonds issued by the central bank.

Accordingly, the model predicts the existence of different interest rate spreads (excess returns) that limit banks' ability to borrow. When the incentive constraint binds and households face limited participation in foreign currency deposits, not only the return on banks' assets exceeds the return on deposits, including the excess return to foreign currency-denominated loans, but also the return on domestic currency-denominated deposits exceeds the return on foreign currency-denominated liabilities. Consequently, when financial frictions are active, the model predicts deviations from the standard UIP condition: banks would be willing to borrow more from households and from international financial markets in foreign currency, while households are unable to engage in frictionless arbitrage of foreign currency-denominated deposit returns.

In this setting, we study the transmission of external shocks to domestic financial conditions by assessing the role of FX intervention operations to "lean against the wind" with respect to exchange rate fluctuations and stabilize the response of interest rate spreads and bank lending. External shocks are transmitted to the domestic economy through changes in the exchange rate, interest rate spreads, and banks' net worth. FX intervention policy is non-neutral when limits to arbitrage are present for banks and households. For example, a persistent commodity boom generates a domestic economic expansion that, among other things, rises commodity exports significantly. Under a free-floating regime, the exchange rate appreciation relaxes the agency problem by increasing banks net worth and intermediation capacity. Hence, after the shock, banks are less exposed to foreign currency liabilities. The latter effect is reinforced by a persistent decline in the banking system currency mismatch that relaxes the financial constraint even more. By the same token, the interest rate spreads of banks' assets over deposits move towards inducing banks to lend more in both currencies. It is noticeable that the persistent exchange rate appreciation increases credit dollarization but reduces deposit dollarization.

When the FX intervention policy is active, the central bank builds FX reserves and allocates central bank riskless bonds to the banking system as a response to commodity booms. Given the binding agency problem, building FX reserves after a persistent increase in commodity prices significantly reduces exchange rate appreciation as well as the responses of currency mismatch and banks' net worth, thereby limiting bank credit growth and the consequent expansion of macroeconomic aggregates such as consumption and investment. Besides exchange rate stabilization and its direct effects on intermediation, our framework implies an additional channel for FX interventions associated with the sterilization process. The associated sterilization operation increases the supply of central bank bonds to be absorbed by banks. The latter generates a crowding-out effect in banks' balance sheets that reduces bank intermediation. Consequently, FX interventions present two potential transmission mechanisms in our framework, the exchange rate smoothing channel and the balance sheet substitution channel. The former channel affects the size of the currency mismatch at the bank level, while the latter works through the availability of bank resources to extend loans.

We take the model to the data to quantify the transmission mechanism of external shocks and the role of FX interventions in mitigating their impact on the domestic economy. We consider not only commodity price shocks as described above, but also shocks to the foreign interest rate and global GDP. This exercise is intended to quantify the differences in the response of the economy to external shocks when FX interventions are activated, compared to exchange rate flexibility. We also conduct a standard welfare exercise to analyze whether FX interventions yield welfare gains in the presence of external shocks.

Recent empirical evidence show that our framework is general enough to be consistent with the experience of many EMEs facing frequent external shocks under a managed exchange rate regime along with banking systems characterized by significant financial dollarization and currency mismatch. On one hand, Levy-Yeyati and Sturzenegger (2016) classify the exchange rate regime of emerging market and advanced economies based on a "de facto" criterion and find that more than half of the countries in their sample adopt a non-floating exchange rate regime. Based on the same criteria, Aguirre et al. (2019) report that none of the countries that have implemented IT since 1991 have always kept a purely floating exchange rate regime. Moreover, periods during which several countries (reaching around 60 percent of them) were non-pure floaters coincide with events related to external fundamentals. On the other hand, Corrales and Imam (2019) examine countries from different regions using the International Financial Statistics database from 2001 to 2016 and report that households maintain 57.5 percent of their deposits in dollars, while for firms, 68.7 percent of their loans are denominated in dollars. Castillo et al. (2019) study 45 emerging market and advanced economies, excluding countries whose central bank issues a reserve currency, and report that around 50 percent of the countries in their sample are classified as dollarized economies.

Our quantitative analysis uses data for the Peruvian economy since it is representative of EMEs under an inflation targeting regime with active FX intervention operations, financial dollarization, and a commodity-exporter small open economy facing external shocks continuously. We consider that using data for several EMEs instead may be misleading since evidence also shows that there is a high degree of heterogeneity in the strategies, instruments, and tactics used to implement FX intervention policies (see Hendrick et al., 2019, and Patel and Cavallino, 2019). Therefore, we calibrate most of the parameters associated with the banking block of the model to replicate some financial steady-state targets for Peru's banking system. The rest of the parameterization is done by matching the impulse responses of the economic model to the impulse responses implied by an SVAR model with block exogeneity under the small open economy assumption.

Quantitatively, our results suggest that, conditional on external shocks, FX intervention operations successfully reduce macroeconomic volatility relative to a free-floating regime. In particular, under a FX intervention regime, the volatility of credit, investment, and output falls by around 82, 65, and 70 percent, respectively, when compared to a flexible exchange rate regime. Then, FX interventions play the role of an external shock absorber. These stability implications are indicative that FX intervention might create significant welfare gains as a response to external shocks. Hence, we use a standard welfare analysis and find that if the central bank does not intervene in the FX market in the face of external shocks, there would be a welfare loss of 6.2 percent in consumption, given the standard parameterization of the Taylor rule for the conventional interest rate instrument.

Furthermore, we explore additional numerical experiments. We recalibrate the steady state of the model economy to be consistent with a lower steady state level for the average currency mismatch of the banking system. We consider a decrease of six percentage points relative to our baseline calibration by targeting a higher foreign interest rate at the steady state. These targets induce banks to be less exposed to potential currency mismatches. Not surprisingly, our results suggest that FX interventions are less effective when the economy is calibrated to be consistent with a lower level of currency mismatch at the steady state since banks are in a less vulnerable initial position with respect to external shocks that produce unexpected depreciations.

Then we relax three assumptions of our basic formulation of the model that may be viewed as strong and restrictive with the aim to study our setting under more general assumptions. First, we consider the case of an economy without financial dollarization where intermediate good producers borrow from banks only in domestic currency and households are not allowed to hold deposits with banks that are denominated in foreign currency. Consequently, banks lend only in domestic currency, while the only source of foreign currency funding for banks comes from borrowing abroad. In the steady state equilibrium banks are more exposed to real exchange rate movements, while non-financial firms as well as households are less exposed to these fluctuations. Our parametrization suggests that when the economy is not financially dollarized, FX intervention operations are still non-neutral but less effective than in the financially dollarized economy in smoothing the response of the exchange rate as well as the response of financial and macroeconomic variables to external shocks.

Second, we relax the limited participation assumption of households with respect to bank deposits denominated in foreign currency by assuming a limiting case of zero transaction costs. Consequently, households' demand for bank deposits in foreign currency is infinitely responsive to arbitrage opportunities, implying that in equilibrium the UIP condition for households holds with a constant premium while the incentive compatibility constraint for banks is still binding. Our simulations show that in this case, the exchange rate smoothing channel of FX interventions is not active, nevertheless the sterilization process associated with the FX intervention operation presents a relatively small effect on financial and macroeconomic variables due to the balance sheet substitution channel. In our model, for FX interventions to affect significantly the real exchange rate and excess returns along with the aggregate equilibrium of the economy, limits to arbitrage between domestic and foreign currency-denominated assets and liabilities must be present for both households and banks.

In the last extension of the model, the severity of the bank's agency problem depends directly on an industry (aggregate) measure of currency mismatch instead than on an individual measure. In this case, banks do not internalize the effects of borrowing and lending in foreign currency on the aggregate currency mismatch of the banking system. As a result, banks are indifferent between borrowing from domestic depositors and from abroad, implying that the standard UIP condition holds without any endogenous risk premium. Notably in this case, even though the incentive constraint for banks binds the response of the real exchange rate to external shocks is the same under FX interventions and exchange rate flexibility. This result differs from Céspedes et al. (2017) and Chang (2019), where FX interventions are irrelevant only when the incentive compatibility constraint does not bind. In this extension, the associated sterilization operation generates negligible real effects for several macroeconomic variables relative to our baseline case. Thus, in terms of macroeconomic variables different from the real exchange rate, FX interventions are less effective in this case since the exchange rate smoothing channel is muted. Our result is due to the indeterminacy of banks' liability composition that occurs when banks do not internalize the effect of currency mismatch over financial constraints. Furthermore, we simulate an exogenous purchase of FX reserves under the last two extensions of the model and find that FX interventions are irrelevant for real exchange rate dynamics even when the incentive compatibility constraint binds.

Finally, we compare the performance of our FX intervention policy with an alternative policy that implements a managed float by using the policy interest rate as the unique monetary instrument. The latter policy is characterized by an extended Taylor rule where the policy interest rate responds not only to inflation and the output gap, but also to deviations of the real exchange rate with respect to its steady state value. Our findings suggest that when the central bank uses the policy rate to smooth exchange rate fluctuations, it leads to exchange rate and financial stabilization at the expense of real destabilization, especially of investment. This result suggest that sterilized FX intervention may be important as an additional independent instrument available to the central bank under certain conditions.

The remainder of the paper is organized as follows. After a brief review of the literature related to FX interventions in macroeconomic models, Section 2 describes the general equilibrium model with a special emphasis in the financial system and the implementation of FX interventions. Section 3 presents the parametrization strategy, including the specification and identification assumptions for the SVAR model. The main results are discussed in Section 4. Section 5 studies the effects of external shocks on some generalizations of our basic formulation of the model. Finally, Section 6 concludes with some final remarks.

Related Literature. Pioneered by Kouri (1976), Branson et al. (1977), and Henderson and Rogoff (1982), the first strand of this literature emphasizes the portfolio balance channel, which indicates that, when domestic and foreign assets are imperfect substitutes, FX intervention is an additional and effective central bank tool. This is because it can change the relative stock of assets and with it the exchange rate risk premium that affects arbitrage possibilities between the rates of return of domestic currency-denominated assets and foreign currency-denominated assets. However, the models built during this stage were characterized by a lack of solid micro-foundations, preventing a rigorous normative analysis. Additional research studies within the portfolio balance approach without micro-foundations are Krugman (1981), Obstfeld (1983), Dornbusch (1980), Branson and Henderson (1985), and Frenkel and Mussa (1985).

Relying on micro-founded general equilibrium models, the second strand of this literature states that FX interventions have no effect on equilibrium prices and quantities. The seminal work using this approach is Backus and Kehoe (1989), which not only studies the effectiveness of this kind of intervention in complete markets, but also considers some types of market incompleteness. It points out that, when portfolio decisions are frictionless, the imperfect substitutability between domestic and foreign assets postulated by the portfolio balance channel is not enough for FX interventions to affect prices and quantities in the general equilibrium. After the publication of this work, academia adopted a pessimistic view with respect to the effectiveness of FX interventions, creating a long-lasting dissonance with policy practice since policymakers have ignored the recommendations from research and have intervened, frequently and intensely, in the FX market.

Recently, there has been a resurgence in academic interest in assessing the relevance of FX interventions based on micro-founded macroeconomic models. In this regard, the portfolio balance approach has experienced a recent comeback in studies such as Kumhof (2010), Gabaix and Maggiori (2015), Liu and Spiegel (2015), Benes et al. (2015), Montoro and Ortiz (2016), Cavallino (2019), and Castillo et al. (2019). Some of these studies rely on a reduced form type of friction while others assume more structure when addressing the relevance of FX interventions. This literature argues that FX intervention can affect the exchange rate when domestic and external assets are imperfect substitutes. In this case, FX intervention increases the relative supply of domestic assets, driving the risk premium up and creating exchange rate depreciation pressures.

A third strand of the literature is the so-called financial intermediation view of FX interventions. The general equilibrium relevance of FX interventions relies on a financial friction of the type associated with the literature on unconventional monetary policy in closed economies. Specifically, this literature assumes that banks face an agency problem that constraints their ability to obtain funds from abroad. Céspedes et al. (2017) and Chang (2019) build models for an open economy with domestic banks subject to occasionally binding collateral constraints and find that FX interventions have an impact on macroeconomic aggregates only when the relevant financial constraint is binding. When financial markets are frictionless, domestic banks are able to accommodate FX interventions by borrowing less or more from domestic depositors as well as from foreign financial markets. In the latter case, the general equilibrium is left undisrupted.

Additionally, Fanelli and Straub (2021) find that including a pecuniary externality in partially segmented domestic and foreign bond markets results in an excessively volatile exchange rate response to capital inflows, thereby making FX interventions desirable.

Empirical evidence on the effectiveness of FX interventions has been particularly difficult to find because of endogeneity problems that make it difficult to identify its effects, especially on the exchange rate. While individual country studies report mixed results on the effectiveness of FX intervention, in general cross-country studies find some effectiveness in curbing financial conditions and exchange rate dynamics (see Ghosh et al., 2018; Villamizar-Villegas and Pérez-Reyna, 2017; and Fratzscher et al., 2018). Recent empirical findings have shed some light on how FX intervention reduces the impact of capital flows on domestic financial conditions. For instance, Blanchard et al. (2015) show that capital flow shocks have significantly smaller effects on exchange rates and capital accounts in countries that intervene in FX markets on a regular basis. According to Hofmann et al. (2019), FX intervention has two mutually reinforcing effects. On one hand, in periods of easing global financial conditions, FX can be used to lean against the increase in bank lending after a dollar appreciation (the risk-taking channel of the exchange rate). On the other hand, there is a "crowding out" effect of bank lending associated with the sterilization process of the FX intervention, which increases the supply of domestic bonds absorbed by banks. The aggregate impact of FX interventions results from the mix of these two effects. By curbing domestic credit, FX intervention will have an impact on the real economy.

2. A General Equilibrium Model

We build a medium-scale small open economy New Keynesian model extended with banks, FX interventions, and a commodity sector. Following ABK, banks are allowed to finance their assets using two kinds of liabilities: domestic deposits and foreign borrowing from international financial markets. Nevertheless, banks lend not only in domestic currency but also in FX. FX intervention is introduced to study the role of this tool in financial intermediation, macroeconomic stabilization, and exchange rate volatility.

The rest of the model follows very closely the standard small open economy New Keynesian framework, with the exception of two main features. First, we introduce an endogenous commodity sector to analyze the effect of commodity booms and busts in domestic financial conditions. The representative commodity producer accumulates its own capital facing standard capital adjustment costs and does not need external funding or any form of borrowing to produce. Second, we assume that intermediate good producers must borrow from banks before producing. In addition, we assume that intermediate good producers demand a bundle of loans consisting of a combination of domestic and foreign currency-denominated loans according to a loan services technology that aggregates both types of loans. Further details about the model are presented below. For the rest of the document, small letters characterize individual variables, while capital letters denote aggregates.

2.1 The Financial System

We follow Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) and introduce a banking sector in an otherwise standard infinite horizon macroeconomic model for a small open economy. In this setting, the representative household consists of a continuum of bankers and workers of measure unity. Workers supply labor and provide labor income to their households. Workers hold deposits with banks along with private securities in the form of equity with intermediate good producers. Domestic bank deposits are denominated in domestic and foreign currency, although the latter is subject to transaction costs. Foreign agents lend to banks in foreign currency and are precluded from lending directly to non-financial firms. All financial contracts between agents are short-term, non-contingent, and thus riskless. An agency problem constraints banks' ability to obtain funds from households and foreigners. The tightness of the financial constraint that banks face depends on a measure of currency mismatch at the individual level. In this section, we focus on bankers, while workers are described in detail in Section 2.3.

Banks. In a given household, each banker member manages a bank until she retires with probability $1-\sigma$. Retired bankers transfer their earnings back to households in the form of dividends and are replaced by an equal number of workers that randomly become bankers. The relative proportion of bankers and workers is kept constant. New bankers receive a fraction ξ of total assets from the household as start-up funds.

Additionally, banks provide funding to producing firms without any financial friction. Hence, the only financially constrained agents in the model are banks due to a moral hazard problem between a bank and its depositors.[5] Domestic and foreign currency-denominated bank

[5] Households face limited participation in asset markets when saving in foreign currency and holding equity. Limited participation appears in terms of a marginal transaction cost for managing sophisticated portfolios.

loans to firms are denoted by l_t and l_t^*, respectively. Bank assets are also made up of central bank bonds (b_t) considered to be the only financial instruments used in the associated sterilization process of any FX intervention. Bank investments are financed by domestic currency-denominated household deposits (d_t), by foreign currency-denominated household deposits ($d_t^{*,h}$), by foreign borrowing ($d_t^{*,f}$), or by using banks' own net worth (n_t). A bank's balance sheet expressed in real terms is

$$l_t + e_t l_t^* + b_t = n_t + d_t + e_t(\overbrace{d_t^{*,h} + d_t^{*,f}}^{d_t^*}) \tag{1}$$

where e_t is the real exchange rate. Table 1 illustrates the typical balance sheet of a bank in the model.

Table 1. Bank's Balance Sheet

Assets	Liabilities
$\boldsymbol{l_t}$	d_t
$\boldsymbol{e_t l_t^*}$	$e_t(d_t^{*,h} + d_t^{*,f})$
$\boldsymbol{b_t}$	n_t

We assume that $d_t^{*,h}$ and $d_t^{*,f}$ are perfect substitutes for bankers and d_t^* denotes total deposits/funding in foreign currency. Net worth is accumulated through retained earnings, and it is defined as the difference between the gross return on assets and the cost of liabilities:

$$n_{t+1} = R_{t+1}^l l_t + R_{t+1}^{l*} e_{t+1} l_t^* + R_{t+1}^b b_t - R_{t+1} d_t - e_{t+1} R_{t+1}^* d_t^* \tag{2}$$

where $\{R_t^b, R_t^l, R_t^{l*}\}$ denote the real gross returns to the bank from central bank bonds, domestic currency-denominated loans, and foreign currency-denominated loans, respectively. Similarly, R_t and R_t^* are the real gross interest rate paid by the bank on domestic and foreign currency-denominated liabilities, respectively.[6]

Agency Problem. With the purpose of limiting banks' ability to raise domestic and foreign funds, we assume that at the beginning of the period, bankers may choose to divert funds from the assets they hold and transfer the proceeds to their own households. If bank managers operate

[6] All real interest rates are ex post. Along these lines, R_t equals $\frac{1+i_{t-1}}{1+\pi_t}$ where i_t is the nominal policy rate.

honestly, then assets will be held until payoffs are realized in the next period and repay their liabilities to creditors (domestic and foreign). On the contrary, if bank managers decide to divert funds, then assets will be secretly channeled away from investment and consumed by their households. In this framework, it is optimal for bank managers to retain earnings until exiting the industry. Bankers' objective is to maximize the expected discounted stream of profits that are transferred back to the household, i.e., its expected terminal wealth, given by

$$V_t = \mathbb{E}_t \left[\sum_{j=1}^{\infty} \Lambda_{t,t+j} \sigma^{j-1} (1-\sigma) n_{t+j} \right]$$

where $\Lambda_{t,t+j}$ is the stochastic discount factor of the representative household from $t+j$ to t and $\mathbb{E}_t[.]$ is the expectation operator conditional on information set at t. Notice that using $\Lambda_{t,t+j}$ to properly discount the stream of bank profits means that households effectively own the banks that their banker members manage. Bank managers will abscond funds if the amount they are capable of diverting exceeds the continuation value of the bank V_t. Accordingly, for creditors to be willing to supply funds to the banker, any financial arrangement between them must satisfy the following incentive constraint:

$$V_t \geq \Theta(x_t)[l_t + \varpi^* e_t l_t^* + \varpi^b b_t] \tag{3}$$

where $\Theta_t(x)$ is assumed to be strictly increasing[7] and x_t is the currency mismatch measure at the bank level defined and discussed below. We assume that some assets are more difficult to divert than others. Specifically, a banker can divert a fraction $\Theta(x_t)$ of domestic currency loans, a fraction $\Theta(x_t)\varpi^*$ of foreign currency loans, and a fraction $\Theta(x_t)\varpi^b$ of the total amount of central banks bonds, where $\varpi^*, \varpi^b \in [0, \infty)$. For instance, whenever $\varpi^b = 0$, bankers cannot divert sterilized bonds and buying them does not tighten the incentive constraint. Therefore, a fraction of the interest rate spread on $b_t$may be arbitraged away, leaving R_t^b lower than R_t^l. In our setting, the three types of assets held by banks do not enter with equal weights into the incentive constraint, reflecting that for some assets the constraint on arbitrage is weaker. We calibrate ϖ^*, and ϖ^b to match the average gross returns for each asset type in the Peruvian economy. In Section 3, we

[7] Specifically, we use the following convex function:

$$\Theta(x) = \theta(1 + \frac{\varkappa}{2} x^2)$$

show that those targets are consistent with the fact that central bank bonds are much harder to divert than loans; i.e., the calibrated ϖ^b is very close to zero. In Section 5 we relax this assumption and assume that all assets enter the incentive constraint with equal weights.

We assume that the banker's ability to divert funds depends on the currency mismatch size at the bank level expressed as a fraction of total assets. In this regard, we define x_t to be

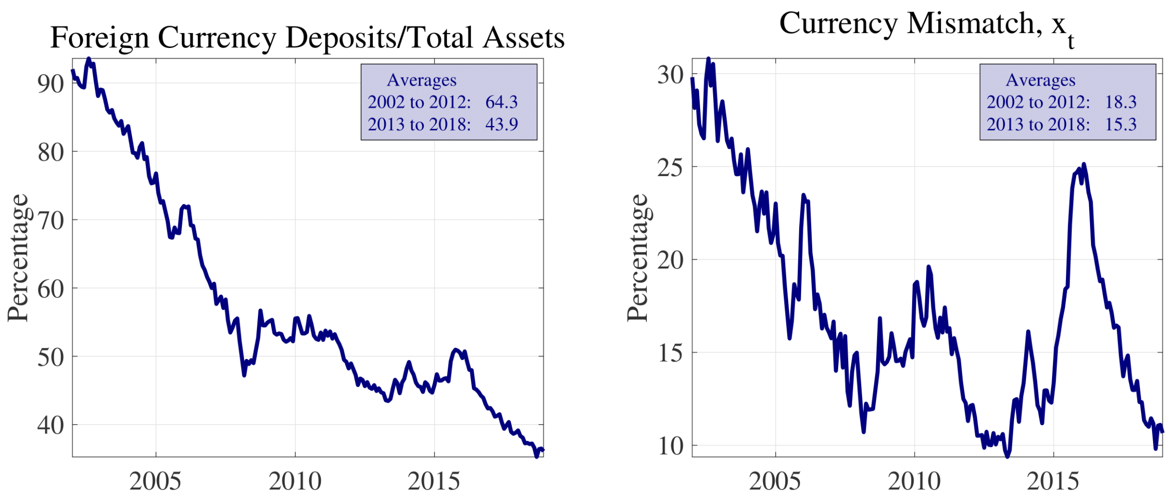
$$x_t = \frac{e_t d_t^* - e_t l_t^*}{l_t + e_t l_t^* + b_t} \tag{4}$$

A higher currency mismatch at the bank level implies that bankers are able to divert a higher fraction of their assets, ultimately increasing the severity of the incentive constraint. In this regard, x_t measures the exposure of the bank's balance sheet to abrupt foreign capital reversals and exchange rate movements. A significant currency mismatch degree in a bank's balance sheet places it in a more vulnerable position with respect to external shocks, particularly shocks generating unexpected depreciations. From this perspective, and as long as the incentive constraint is binding, an increase in x_t will require an increase in V_t to keep domestic depositors and foreign lenders willing to continue lending funds to a bank. In the basic formulation of the model, we assume that x_t is internalized by each bank. In Section 5, we assume that x_t is external to an individual bank representing an aggregate currency mismatch measure of the banking system.

Figure 1 plots the empirical counterpart for both the evolution of foreign currency liabilities and the currency mismatch level of Peru's banking system. The latter is also known as the FX spot or countable net position of a bank without considering FX derivatives.[8] Foreign currency deposits, including external credit lines (bank's foreign borrowing), expressed as a fraction of total assets, have been steadily decreasing since 2001, from an average of 64.3 percent during 2001-2012 to an average of 43.9 percent from 2009 to 2018. This is also the case for our empirical measure of currency mismatch, which likewise shows a markedly decreasing trend from 2001 to 2012 with an average of 18.3 percent. From 2009 to 2018, it has been fluctuating around 15.3 percent without showing a clear trend.

[8] Appendix A fully describes the strategy followed to build the bank's balance sheet and other empirical counterparts of the model.

Figure 1. Foreign Deposits and Currency Mismatch

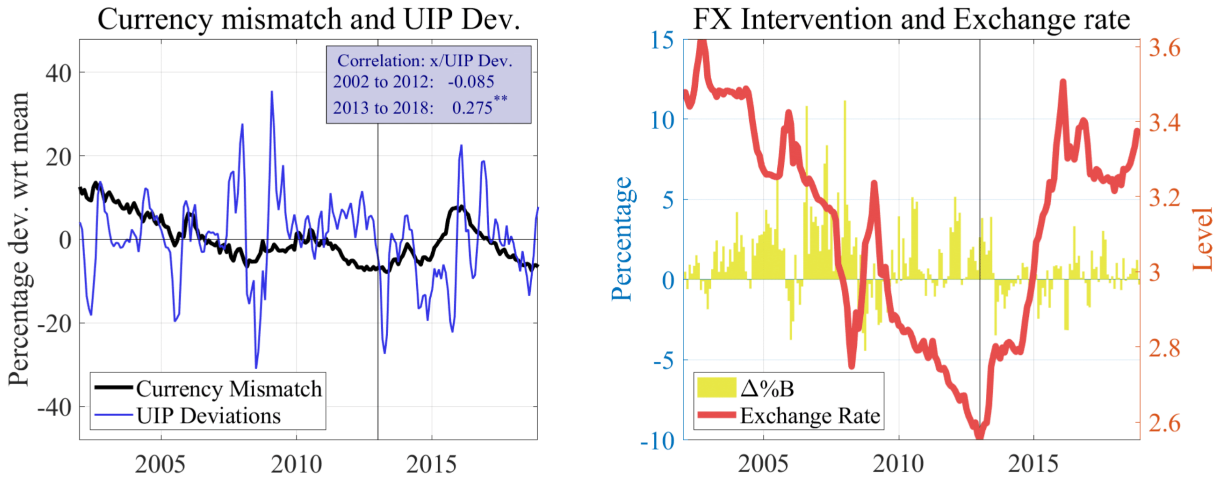


On the other hand, Figure 2 plots the evolution of the empirical counterpart of the currency mismatch level of Peru's banking system compared with empirically calculated UIP deviations from January 2002 to December 2018. From the point of view of the banking system, UIP deviations are defined as the interest rate spread of domestic currency deposits relative to bank's foreign borrowing, $\mathbb{E}_t[R_t - e_{t+1}R_t^*/e_t]$.[9] Although the dynamics of the empirical currency mismatch may respond to different economic fundamentals, it exhibits a positive correlation with UIP deviations. In our model, this correlation comes from the assumption that the currency mismatch at the bank level, determines the severity of the incentive constraint faced by banks, i.e., $\partial_x \Theta(x) > 0$ for all $x > 0$. At a micro level, in a recent paper di Giovanni et al. (2017) present empirical support for this co-movement in emerging markets by exploiting Turkish banking data.

Moreover, in our model, the FX intervention policy affects the dynamics of the real exchange rate, the currency mismatch as well as the magnitude and persistence of UIP deviations, ultimately reducing the aforementioned correlation and its corresponding volatility. In line with the theoretical predictions of our model, Figure 2 reports that the correlation between the currency mismatch and UIP deviations is positively strong during a period of moderate FX interventions (from 2013 to 2018) but weak under a strong FX intervention period (from 2002 to 2012).

[9] Since Peru is a typical commodity-exporting emerging market economy under an inflation targeting regime with active FX intervention policy and financial dollarization, we mainly use Peruvian data for our quantitative analysis.

Figure 2. Currency Mismatch, FX intervention and UIP Deviation



Bank's Recursive problem. Given a function $\Theta(x)$, a vector of interest rates, government policies, and n_t (state variable), each bank chooses its balance sheet components $(l_t, l_t^*, b_t, d_t, d_t^*)$ to maximize the franchise value:

$$V_t = \max_{l_t, l_t^*, b_t, d_t, d_t^*} \mathbb{E}_t[\Lambda_{t,t+1}\{(1-\sigma)n_{t+1} + \sigma V_{t+1}\}]$$

subject to (1), (2), (3), and (4).

A bank's objective function as well as its balance sheet and the incentive constraint it faces, can be expressed as a fraction of net worth. Moreover, using the definition of x_t, a bank's problem can be written in terms of choosing each of the assets it holds as a fraction of net worth together with the optimal size of its currency mismatch x_t. Consequently, the bank's problem is to choose $(\phi_t, \phi_t^*, \phi_t^b, x_t)$ to maximize its value as a fraction of net worth:

$$\psi_t = \max_{\phi_t^l, \phi_t^{l*} \phi_t^b, x_t} \mu_t^l \phi_t^l + (\mu_t^{l*} + \mu_t^{d*})\phi_t^{l*} + \mu_t^b \phi_t^b + \mu_t^{d*}(\phi_t^l + \phi_t^{l*} + \phi_t^b)x_t + \upsilon_t \tag{5}$$

subject to:

$$\psi_t \geq \Theta(x_t)[\phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b] \tag{6}$$

where $\psi_t = \frac{V_t}{n_t}$, $\phi_t = \frac{l_t}{n_t}$, $\phi_t^* = \frac{e_t l_t^*}{n_t}$, $\phi_t^b = \frac{b_t}{n_t}$, $\upsilon_t = \mathbb{E}_t[\Omega_{t+1} R_{t+1}]$, and

$$\mu_t^l \quad = \mathbb{E}_t[\Omega_{t+1}(R_{t+1}^l - R_{t+1})]; \mu_t^{l*} = \mathbb{E}_t[\Omega_{t+1}(\frac{e_{t+1}}{e_t} R_{t+1}^{l*} - R_{t+1})]$$
$$\mu_t^b \quad = \mathbb{E}_t[\Omega_{t+1}(R_{t+1}^b - R_{t+1})]; \mu_t^{d*} = \mathbb{E}_t[\Omega_{t+1}(R_{t+1} - \frac{e_{t+1}}{e_t} R_{t+1}^*)]$$

Ω_{t+1} is the shadow value of a unit of net worth to the bank at $t+1$, given by

$$\Omega_{t+1} = \Lambda_{t,t+1}(1 - \sigma + \sigma\psi_{t+1})$$

Let λ_t^b be the Lagrangian multiplier for the incentive constraint faced by the bank, eq. (6). Then, the first order conditions are characterized by the slackness condition associated with eq. (6) and:[10]

$$\mu_t^l + \mu_t^{d*} x_t = \frac{\lambda_t^b}{1 + \lambda_t^b} \Theta(x_t) \tag{7}$$

$$\mu_t^{l*} + \mu_t^{d*}(1 + x_t) = \frac{\lambda_t^b}{1 + \lambda_t^b} \varpi^* \Theta(x_t) \tag{8}$$

$$\mu_t^b + \mu_t^{d*} x_t = \frac{\lambda_t^b}{1 + \lambda_t^b} \varpi^b \Theta(x_t) \tag{9}$$

$$\mu_t^{d*}(\phi_t^l + \phi_t^{l*} + \phi_t^b) = \frac{\lambda_t^b}{1 + \lambda_t^b} (\phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b) \frac{\partial \Theta(x_t)}{\partial x} \tag{10}$$

When the incentive constraint is not binding, then $\lambda_t^b = 0$, the discounted excess returns or interest rate spreads are zero. Consequently, under this equilibrium, financial markets are frictionless, implying that the standard arbitrage condition holds: banks will acquire assets to the point where the discounted return on each asset equals the discounted cost of deposits (i.e., $\mu_t^l = \mu_t^{l*} = \mu_t^b = 0$). In addition, there is no cost advantage of foreign borrowing over domestic deposits (i.e., $\mu_t^{d*} = 0$, the UIP conditions holds).

When the incentive constraint is binding, $\lambda_t^b > 0$, banks are restricted in obtaining funds from creditors. In this context, limits to arbitrage emerge in equilibrium, leading to interest rate spreads. It is important to highlight that excess returns increase depending on how tightly the incentive constraint binds. The latter is measured by λ_t^b and ultimately depends on x_t. The intuition behind the above first-order conditions is that banks invest in each asset to the point where the

[10] A complete derivation of the bank's optimality conditions is presented in Appendix C.1.

marginal benefit of acquiring an additional unit of each asset is equal to its marginal cost. The marginal benefit of each asset is composed of its own discounted excess value and the excess value associated with the advantage cost of funding it via foreign borrowing, which is ultimately influenced by the size of the currency mismatch.[11] For instance, a fraction x_t of an extra unit of l_t or b_t is funded by d_t^*. Similarly, a portion $1 + x_t$ of an additional investment in l_t^* is financed by d_t^*; i.e., banks use more foreign currency funds and less home deposits per unit of foreign currency loans. On the other hand, the marginal cost associated with each asset is given by the marginal cost of tightening the incentive constraint times the total share of the asset that the bank may actually divert.

Limits to arbitrage emerge from the restriction that the incentive constraint places on the size of a bank's portfolio relative to its net worth. A form of leverage ratio for a bank can be obtained by combining eq. (5), eq. (6), and the above first order conditions,

$$\Phi_t n_t \geq l_t + \varpi^* e_t l_t^* + \varpi^b b_t \tag{11}$$

$$\Phi_t = \frac{v_t}{\Theta(x_t) - (\mu_t^l + \mu_t^{d*} x_t)} \tag{12}$$

Gertler and Karadi (2013) argued that Φ_t can be interpreted as the maximum ratio of weighted assets to net worth that a bank may hold without violating the incentive constraint. The weight applied to each asset is the proportion of the asset that the bank is able to divert.

When the incentive constraint binds, the weighted leverage ratio Φ_t is increasing in two factors: i) the savings of deposit costs from another unit of net worth given by v_t; and ii) the discounted marginal benefit of lending in domestic currency. As discussed in Gertler et al. (2012), both factors raise the value of a bank, thereby making its creditors willing to lend more. The leverage ratio also varies inversely with exchange risk perceptions ultimately associated with fluctuations in x_t: whenever the currency mismatch rises, bankers are more exposed to real

[11] Note that the marginal benefit for each asset can be rewritten in terms of interest rate spreads as

$$\begin{aligned} \mu_t^l + \mu_t^{d*} x_t &= \mathbb{E}_t[\Omega_{t+1}(R_{t+1}^l - \{\frac{e_{t+1}}{e_t} R_{t+1}^* x_t + R_{t+1}(1 - x_t)\})] \\ \mu_t^b + \mu_t^{d*} x_t &= \mathbb{E}_t[\Omega_{t+1}(R_{t+1}^b - \{\frac{e_{t+1}}{e_t} R_{t+1}^* x_t + R_{t+1}(1 - x_t)\})] \\ \mu_t^{l*} + \mu_t^{d*}(1 + x_t) &= \mathbb{E}_t[\Omega_{t+1}(R_{t+1}^{l*} - \{\frac{e_{t+1}}{e_t} R_{t+1}^*(1 + x_t) + R_{t+1}(-x_t)\})] \end{aligned}$$

Then, it is clear that x_t directly influences the fraction of each asset financed by foreign currency borrowing.

exchange movements and its creditors restrict external funding. Notice that in a closed economy setting, μ_t^{d*} is zero and Φ_t constant. In this case, eq. (12) converges to the setup for a bank's leverage ratio proposed by Gertler and Karadi (2013).

The leverage ratio can be expressed as a collateral constraint consistent with Kiyotaki and Moore (1997) as follows:

$$l_t \leq \theta_t n_t \qquad \text{and} \qquad \theta_t = \Phi_t - \varpi^* \phi_t^* - \varpi^b \phi_t^b$$

where $\phi_t^* = \frac{e_t l_t}{n_t}$ and $\phi_t^b = \frac{b_t}{n_t}$. Recently, Céspedes et al. (2017) and Chang (2019) use similar collateral constraints to capture foreign debt limits faced by EME domestic banks. However, in our more general framework, θ_t is not a parameter but an endogenous variable that depends on a currency mismatch measure at the bank level. In our setting, similar collateral constraints for l_t^* and b_t can be obtained straightforwardly.

To wrap out, in our model, the non-neutrality result of FX intervention policy for the general equilibrium allocation is a consequence of the following deviation of the UIP equation:

$$\mathbb{E}_t \Omega_{t+1}(R_{t+1} - \frac{e_{t+1}}{e_t} R_{t+1}^*) = \frac{\lambda_t^b}{1+\lambda_t^b}(\frac{l_t + \varpi^* e_t l_t^* + \varpi^b b_t}{l_t + e_t l_t^* + b_t}) \frac{d\Theta(x_t)}{dx_t} \qquad (13)$$

For FX interventions to affect significantly real exchange rate dynamics, limits to arbitrage between domestic and foreign currency-denominated assets and liabilities must be present, i.e., $\lambda_t^b > 0$. However, this is only a necessary condition. If $\lambda_t^b > 0$, but banks do not internalize the effects of the currency mismatch on the severity of the agency problem (i.e., Θ depends on an aggregate measure of currency mismatch implying that $\frac{d\Theta(x)}{dx} = 0$), then expected UIP deviations are equal to zero and FX interventions barely affect real exchange rate dynamics. Additionally, if households are able to engage in frictionless arbitrage between foreign currency and domestic currency bank deposits, FX interventions are neutral with respect to exchange rate dynamics. Finally, it is worth mentioning that, even with $\frac{d\Theta(x)}{dx} = 0$, FX operations could affect the macroeconomic allocation through its effects on the bank's balance sheet, as long as $\lambda_t^b > 0$. The relevance of these assumptions on the effectiveness of FX interventions are explored in more detail in Section 5 below.

2.2 *The Central Bank and FX Interventions*

The related literature on FX intervention (for example, Chang, 2019) agrees in defining it as the following situation: whenever a central bank buys or sells FX and at the same time also buys or sells an equivalent amount of domestic currency-denominated securities. Under this policy, the central bank's net credit position changes. Without sterilization, buying or selling FX would directly affect the supply of domestic liquidity. The latter implies difficulties in meeting the central bank's interbank interest rate target, which ultimately is determined by a Taylor rule. Nevertheless, there is less agreement in the literature about the implementation of the sterilization leg of FX interventions. This reflects differences in FX intervention practices among central banks.

In our framework, the sterilization operations associated with an FX intervention are implemented by changing the supply of central bank bonds in the banking system. Recall that central bank bonds are riskless one-period bonds issued by the monetary authority. Accordingly, FX intervention denotes the following: if the central bank buys (sells) FX, such as dollars, from (to) the domestic banking system, a simultaneous raise (fall) in official FX reserves would occur. At the same time, the central bank will completely offset the effect on domestic liquidity by issuing (retiring) central bank bonds to (from) the banking system. The central bank's balance sheet is given by

$$B_t = e_t F_t \tag{14}$$

where B_t denotes central bank bonds and F_t official FX reserves. Notice that eq. (14) serves both as a sterilization rule and as accounting identity for the central bank's balance sheet. In this setting, FX interventions induce the central bank to produce operational losses or a quasi-fiscal deficit, since it is assumed that official FX reserves are invested abroad at the foreign interest rate R_t^*, while central bank bonds pay R_t^b. As long as, $R_t^b > R_t^*$, the central bank produces operational losses associated with the sterilization process, which ultimately represent the fiscal costs of FX interventions. We assume that central bank's operational losses are transferred to the central government and financed through lump sum taxes on households. Then, the central bank's quasi-fiscal deficit is:

$$CB_t = (R_t^b - \frac{e_t}{e_{t-1}} R_t^*) B_{t-1} \tag{15}$$

Furthermore, in addition to the standard interest rate policy rule, the central bank implements the following FX intervention rule written in terms of the supply of central bank bonds responding to exchange rate deviations from its steady-state value:

$$lnB_t = lnB - \upsilon_e(lne_t - lne) \tag{16}$$

where $\upsilon_e \geq 0$ measures the response of FX interventions to deviations of the real exchange rate with respect to its steady state value. The steady state level of central bank bonds is denoted by B. Under this rule, the central bank sells official FX reserves in response to a real depreciation (i.e., whenever the real exchange rate is above its steady state value). As mentioned before, the counterpart of selling reserves is to withdraw central bank bonds from banks' balance sheet, eq. (14). Consequently, FX interventions present two potential transmission mechanisms in our framework: i) when selling official FX reserves to the banking system, the exchange rate is stabilized; and ii) when sterilizing the effect on domestic liquidity, the central bank frees resources from domestic banks to extend additional loans to firms. Moreover, the exchange rate stabilization effect potentially affects the size of the currency mismatch size at the bank level. For instance, ceteris paribus, stabilizing a depreciation pressure on the exchange rate may lead to reducing the currency mismatch size at the bank level. If this is the case, the incentive constraint (more specifically, its degree of tightening) may be relaxed even further, thereby further stimulating domestic financial conditions.

One key aspect of our model is that FX interventions are relevant for determining the general equilibrium allocation only when the incentive constraint binds, as in Céspedes et al. (2017) and Chang (2019). Whenever the incentive constraint is not binding, financial markets are frictionless, meaning there is no leverage constraint for banks nor interest rate spreads. Therefore, balance sheet policies such as FX interventions are irrelevant, since the size and composition of balance sheets, for both the banking system and the central bank, do not matter for equilibrium. In particular, under frictionless financial markets the sterilization process associated with FX interventions does not have real effects: the exchange rate, as well as domestic financial conditions, are determined without any consideration of balance sheets. More important, in our framework, and in contrast with Chang (2019), domestic banks can accommodate the central bank's FX reserve accumulation during "normal" times (non-binding incentive constraint) by increasing domestic deposits, foreign borrowing, or both, since banks are indifferent between domestic currency and foreign currency funding. Therefore, when the incentive constraint is not binding and the central bank accumulates FX reserves it does not necessarily mean that banks will end up more exposed to foreign currency-denominated liabilities. Furthermore, in Section 5, we consider an extension of our baseline model where banks take as given fluctuations in x_t. In this case, banks consider domestic deposits and foreign borrowing as perfect substitutes, the UIP condition holds with equality and FX interventions are irrelevant for exchange rate dynamics even though the incentive constraint binds.

We consider that for EMEs, financial constraints are always binding, even in "normal" times. The difference between normal times and a financial crisis is how tight financial constraints bite. In our framework, the degree of financial constraint tightening depends on the currency mismatch size in banks' balance sheets, which ultimately responds to external shocks. In this context, FX interventions are meant to be an additional central bank instrument aimed to smooth the response of domestic financial conditions to external shocks via exchange rate stabilization.

2.3 Households

Workers supply labor and take labor income to their household. Households use labor income and profits from firm ownership to consume non-commodity goods, save by holding private securities issued by intermediate good producers along with bank deposits. As already mentioned, bank deposits by households are denominated in domestic and foreign currency. We assume that households face increasing transactions costs when holding equity along with foreign currency-denominated bank deposits. The latter assumption prevents frictionless arbitrage due to limited ability to manage sophisticated portfolios. Finally, in line with standard literature on financial and labor market frictions, it is assumed that within each household there is perfect consumption insurance to keep the representative agent assumption. Following Miao and Wang (2010) and Gertler et al. (2012), households' preference structure is

$$\mathbb{W} = (1-\beta)\mathbb{E}_t\left[\sum_{j=0}^{\infty}\beta^j \frac{1}{1-\gamma}\left(C_{t+j} - \mathcal{H}C_{t+j-1} - \frac{\zeta_0}{1+\zeta}H_{t+j}^{1+\zeta}\right)^{1-\gamma}\right] \tag{17}$$

where C_t is consumption and $H_t$is the labor effort in terms of hours worked. The subjective discount factor is given by $\beta \in (0,1)$, $\gamma > 0$, which measures the elasticity of intertemporal substitution, while ζ_0 controls the dis-utility of labor. Additionally, the Frisch elasticity is mainly determined by the interaction of $\zeta > 0$ and the degree of internal habit formation, $\mathcal{H} \in [0,1)$. For instance, if there is no habit formation (i.e., $\mathcal{H} = 0$), this specification abstracts from wealth effects on labor supply as in Greenwood et al. (1988), and the Frisch elasticity is $1/\zeta$.[12]

Bank deposits are assumed to be one-period riskless real assets that pay a gross real return of R_t from period $t-1$ to t. Let D_t and $D_t^{*,h}$ be the total quantity of domestic and foreign currency-denominated deposits, respectively. The amount of new equity acquired by the household is $\mathcal{S}_t$, while w_t denotes the real wage, R_t^{knc} the return on equity, Π_t is net payouts to the household from the ownership of both financial and non-financial firms and T_t denotes the lump-sum taxes needed to finance the central bank's quasifiscal deficit. Hence, the household budget constraint is written as

$$\begin{aligned} C_t + D_t + e_t\left[D_t^{*,h} + \frac{\kappa_{D*}}{2}\left(D_t^{*,h} - \overline{D}^{*,h}\right)^2\right] + \left[\mathcal{S}_t + \frac{\kappa_S}{2}\left(\mathcal{S}_t - \overline{\mathcal{S}}\right)^2\right] + T_t \\ = w_t H_t + \Pi_t + R_t D_{t-1} + R_t^* e_t D_{t-1}^{*,h} + R_t^{knc} \mathcal{S}_{t-1} \end{aligned} \quad (18)$$

where $(\kappa_{D*}, \overline{D}^{*,h})$ and $(\kappa_S, \overline{\mathcal{S}})$ are parameters that control the transaction costs for $D_t^{*,h}$ and $\mathcal{S}_t$, respectively. Accordingly, $\overline{D}^{*,h}$ and $\mathcal{S}$ correspond to the frictionless capacity level for each asset. Consider the case where the marginal transaction cost is infinity. Then, households will hold the respective frictionless value of each asset, which is fully unresponsive to arbitrage opportunities. Notice that Π_t includes the net transfer to household members that become bankers at the beginning of the period, as it is written as:

$$\Pi_t = \text{Profits} + \text{Net worth from retiring bankers} - \text{Bankers' start-up funds}$$

Hence, the representative worker chooses consumption, labor supply, and bank deposits to maximize eq. (17) subject to eq. (1). Let u_{ct} denote the marginal utility of consumption and $\Lambda_{t,t+1}$ the household's stochastic discount factor; then, a household's first order conditions for labor supply and consumption/saving decisions are

$$\mathbb{E}_t u_{ct} w_t = \zeta_0 H_t^{\zeta}\left(C_t - \mathcal{H}C_{t-1} - \frac{\zeta_0}{1+\zeta}H_t^{1+\zeta}\right)^{-\gamma} \quad (19)$$

$$1 = \mathbb{E}_t\left[R_{t+1}\Lambda_{t,t+1}\right] \quad (20)$$

[12] For a complete examination of the labor supply function in the general case $\mathcal{H} \in [0,1)$, see Appendix C.2.

$$D_t^{*,h} = \overline{D}^{*,h} + \frac{\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{e_{t+1}}{e_t} R_{t+1}^{*} - R_{t+1}\right)\right]}{\kappa_{D*}} \tag{21}$$

$$\mathcal{S}_t = \overline{\mathcal{S}} + \frac{\mathbb{E}_t\left[\Lambda_{t,t+1}\left(R_{t+1}^{knc} - R_{t+1}\right)\right]}{\kappa_S} \tag{22}$$

with

$$u_{ct} = (C_t - \mathcal{H}C_{t-1} - \frac{\zeta_0}{1+\zeta} H_t^{1+\zeta})^{-\gamma} - \mathcal{H}\beta\mathbb{E}_t(C_{t+1} - \mathcal{H}C_t - \frac{\zeta_0}{1+\zeta} H_{t+1}^{1+\zeta})^{-\gamma}$$

$$\Lambda_{t,t+1} = \beta \frac{u_{c,t+1}}{u_{ct}}$$

The optimal demand for private securities and foreign currency-denominated bank deposits (eq. (21) and eq. (22), respectively) is increasing in the excess return of each asset but relative to the parameter that governs the marginal transaction cost. Notice that if the marginal transaction costs disappear (i.e., κ_{D*} and κ_S go to zero), households are able to engage in complete arbitrage and excess returns will tend to be constant. On the contrary, when the marginal transaction costs are infinite, the demands for $D_t^{*,h}$and $\mathcal{S}$are completely unresponsive to excess returns and are given by $\overline{D}^{,h}$ and $\overline{\mathcal{S}}$, respectively.

Finally, when households' demand for bank deposits denominated in foreign currency differs from its frictionless level, endogenous deviations from the UIP condition emerge in equilibrium. Bear in mind, that a similar equation was obtained from banks' first order conditions whenever their incentive constraint binds. Therefore, when the incentive constraint for banks is binding and households are unable to engage in complete arbitrage, FX interventions are not neutral. However, if households' demand for bank deposits in foreign currency is infinitely responsive to arbitrage opportunities (i.e., transactions costs become increasingly smaller) the effect of FX interventions on exchange rate dynamics is neutralized even though banks' incentive constraint still binds.

2.4 The Production Sector

There are four types of non-financial firms making up the production side of the model economy: i) non-commodity final good producers, ii) intermediate good producers, iii) capital good producers and iv) the commodity production sector, which takes global commodity prices and external demand as given.

Non-Commodity Final Good Producers. Final goods in the non-commodity sector are produced under perfect competition and using a variety of differentiated intermediate goods y_{jt}^{nc}, with $j \in [0,1]$, according to the following constant returns to scale technology

$$Y_t^{nc} = \left(\int_0^1 y_{jt}^{nc\,\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} \tag{23}$$

where $\eta > 1$ is the elasticity of substitution across goods. The representative firm chooses y_{jt}^{nc} to maximize profits subject to the production function eq. (23) with profits given by:

$$P_t^{nc} Y_t^{nc} - \int_0^1 p_{jt}^{nc} y_{jt}^{nc} dj,$$

The first-order conditions for the jth input are

$$y_{jt}^{nc} = \left(\frac{p_{jt}^{nc}}{P_t^{nc}} \right)^{-\eta} Y_t^{nc}$$

$$P_t^{nc} = \left(\int_0^1 p_{jt}^{nc\,1-\eta} dj \right)^{\frac{1}{1-\eta}}$$

The final homogeneous good can be used either for consumption or to produce capital goods. In addition, part of the final good production is exported for foreign consumption.

Intermediate Good Producers. There is a continuum of monopolistically competitive firms, indexed by $j \in (0,1)$, producing differentiated intermediate goods that are sold to final good producers. Each firm manufactures a single variety, face nominal rigidities in the form of price adjustment costs as in Rotemberg (1982) and pay for their capital expenditures in advance of production with funds borrowed from banks. Each intermediate good producer operates the following constant return to scale technology with three inputs: capital k_{t-1}^{nc}, imported goods m_t, and labor l_t

$$y_{jt}^{nc} = A^{nc} \left(\frac{k_{j,t-1}^{nc}}{\alpha_k} \right)^{\alpha_k} \left(\frac{m_{jt}}{\alpha_m} \right)^{\alpha_m} \left(\frac{h_{jt}}{1 - \alpha_k - \alpha_m} \right)^{1-\alpha_k-\alpha_m} \tag{24}$$

where $\alpha_k > 0$, $\alpha_m > 0$, and $\alpha_k + \alpha_m \in (0,1)$, and A^{nc} denotes the total factor productivity level of the representative intermediate good producer.

We assume that intermediate good producers issue equity, $\mathcal{S}_{j,t}$, to domestic households and borrow from banks in order to acquire capital for production. After obtaining funds, each intermediate good producer buys capital from capital good producers at a unitary price q_t^{nc}. Furthermore, in order to reflect the presence of credit dollarization in some EMEs and the fact that partially dollarized economies might be more vulnerable to external shocks, we assume that an intermediate good producer needs a combination of domestic and foreign currency-denominated loans to buy capital. The combination of both types of loans is achieved assuming a Cobb-Douglas technology that yields a unit measure of disposable funds, $\mathcal{F}_{j,t}$ or loan services. Thus, the loan bundle that an intermediate good producer needs to buy the capital good is the following:

$$\mathcal{F}_{j,t} = A^e l_{j,t}^{1-\delta f} (e_t l_{j,t}^*)^{\delta f} \tag{25}$$

where A^e is the productivity level for aggregate loan services, $l_{j,t}$ and $l_{j,t}^*$ denote domestic and foreign currency-denominated bank loans respectively and the parameter δ^f controls for the degree of credit dollarization in the economy. Finally, at the end of the period, intermediate good producers sell the undepreciated capital, $\lambda_{nc} k_{j,t-1}^{nc}$, to capital good producers.

First order conditions for intermediate good producers are presented in three groups,[13] each associated with the following production stages: i) cost minimization, ii) borrowing from banks and issuing equity to households, and iii) price setting. The cost minimization stage yields the standard conditional demands for each input:

$$z_t = \alpha_k mc_t \frac{y_{jt}^{nc}}{k_{j,t-1}^{nc}} \tag{26}$$

$$e_t = \alpha_m mc_t \frac{y_{jt}^{nc}}{m_{j,t}} \tag{27}$$

$$mc_t = \frac{1}{A_t^{nc}} z_t^{\alpha_k} e_t^{\alpha_m} w_t^{1-\alpha_k-\alpha_m} \tag{28}$$

The borrowing stage is characterized by a non-arbitrage condition that defines the return on capital (see eq. (29) below) and real loan demands in domestic and foreign currency (eq. (30) and eq. (31)):

[13] See Appendix C.3 for a detail derivation of the following equations.

$$R_t^k = \frac{z_t + \lambda_{nc} q_t^{nc}}{q_{t-1}^{nc}} \tag{29}$$

$$l_{j,t} = (1-\delta^f)\left(\frac{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^k}{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^l}\right)\mathcal{F}_{j,t} \tag{30}$$

$$e_t l_{j,t}^* = \delta^f \left(\frac{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^k}{\mathbb{E}_t \Lambda_{t,t+1} \frac{e_{t+1}}{e_t} R_{t+1}^{l*}}\right)\mathcal{F}_{jt} \tag{31}$$

$$q_t^{nc} k_{j,t}^{nc} = \mathcal{S}_{j,t} + \mathcal{F}_{j,t} \tag{32}$$

In equilibrium, issuing equity and borrowing from banks are considered to be perfect substitutes to intermediate good producers, since both generate equal expected real costs. The demand schedules for domestic and foreign currency loans depend directly on the expected return on capital as well as on the current value of acquired capital by each firm and inversely on the expected interest rate cost of each type of credit. Therefore, in equilibrium the degree of credit dollarization, given by $\frac{eL_t^*}{L_t + eL_t^*}$ where e is the steady-state real exchange rate, is an endogenous variable that depends on domestic financial conditions. The parameter δ^f determines if intermediate good producers need to borrow in foreign currency from banks. Whenever $\delta^f = 0$, the demand for foreign currency loans is zero and banks' balance sheet is such that there is no asset dollarization (see Section 5).

Finally, the price setting stage is characterized by the following New Keynesian Phillips curve:

$$(1+\pi_t)\pi_t = \frac{1}{\kappa}(1-\eta+\eta mc_t) + \mathbb{E}_t\left[\Lambda_{t,t+1}(1+\pi_{t+1})\pi_{t+1}\frac{Y_{t+1}^{nc}}{Y_t^{nc}}\right] \tag{33}$$

Capital Good Producers. There is a continuum of capital producers operating in a competitive market. Each capital good producer uses final goods as inputs in the form of non-commodity investments, as well as the undepreciated capital bought from intermediate good producers. New capital is produced using the following technology:

$$K_t^{nc} = I_t^{nc} + \lambda_{nc} K_{t-1}^{nc} \tag{34}$$

where $K_t^{nc}$is sold to intermediate good producers at the price q_t^{nc}. Producing capital implies an additional cost of $\Phi^{nc}(\frac{I_t^{nc}}{I^{nc}})I_t^{nc}$, which represents the adjustment cost of investment. The latter assumption is introduced to replicate some empirical moments.[14] Given that households own the capital good firm, the objective of a capital producer is to choose $\{I_{t+j}^{nc}\}_{j\geq 0}$ to solve:

$$\mathbb{E}_t\left[\sum_{j=0}^{\infty}\Lambda_{t,t+j}\left(q_{t+j}^{nc}I_{t+j}^{nc}-\left[1+\Phi^{nc}\left(\frac{I_{t+j}^{nc}}{I^{nc}}\right)\right]I_{t+j}^{nc}\right)\right]$$

Profit maximization implies that the price of capital goods is equal to the marginal cost of investment good production as follows:

$$q_t^{nc}=1+\Phi^{nc}\left(\frac{I_t^{nc}}{I^{nc}}\right)+\left(\frac{I_t^{nc}}{I^{nc}}\right)\partial\Phi_t^{nc} \tag{35}$$

where $\partial\Phi_t^{nc}$denotes the first derivative of $\Phi^{nc}(.)$ evaluated at $\frac{I_t^{nc}}{I^{nc}}$.

Commodity Sector. Commodity price movements play a major role in commodity exporting EMEs. Terms-of-trade fluctuations constitute an important driver of business cycle fluctuations in EMEs, for example, episodes of persistently high commodity prices, generate significant economic expansions as well as credit booms.[15]

We introduce a commodity sector with a representative firm that produces a homogeneous commodity good taking global commodity prices and external demand as given. We assume this firm is owned by both foreign and domestic agents. Commodity production is entirely exported abroad and is conducted using capital specific to this sector as the only input. Capital is acquired directly from final good producers and is used to produce commodity-sector capital without any lending from the banking system. Technology in this sector is

$$Y_t^c=A^c(K_{t-1}^c)^{\alpha_c} \tag{36}$$

where Y_t^c is the commodity production, K_t^c is the specific capital for the commodity sector, and A^c is the productivity level in this sector. We assume the commodity firm's ownership is divided

[14] The function $\Phi^{nc}(.)$ must satisfy the following restrictions: $\Phi^{nc}(1)=\Phi^{nc\prime}(1)=0$ and $\Phi^{nc\prime\prime}(.)>0$.
[15] For empirical evidence on this fact, see Fornero et al. (2015), Shousha (2016), Fernández et al. (2017), García-Cicco et al. (2017), and Drechsel and Tenreyro (2018).

between domestic and foreign shareholders, but the domestic household owns a higher fraction of it. Specifically, domestic households own a fraction χ^c of the total firm's value while foreign families own $(1-\chi^c)$ of it. Moreover, we assume that commodity firms must pay a fraction τ^c of their profits as domestic government taxes.

The representative commodity producer faces investment adjustment costs of $\Phi^c\left(\frac{I_t^c}{I^c}\right)$. Thus, capital accumulation is done through the following equation:

$$K_t^c = I_t^c + \lambda_c K_{t-1}^c \tag{37}$$

The representative producer problem in the commodity sector is to choose $\{K_{t+s}^c\}_{s\geq 0}$ and $\{I_{t+s}^c\}_{s\geq 0}$ to maximize[16]

$$\sum_{s=0}^{\infty} \Lambda_{t,t+s}(1-\tau^c)\left(p_{t+s}^c A^c (K_{t+s-1}^c)^{\alpha_c} - \left[1+\Phi^c\left(\frac{I_{t+s}^c}{I^c}\right)\right] I_{t+s}^c\right)$$

subject to eq. (36). The first order conditions for the above problem are given by

$$q_t^c = 1 + \Phi^c\left(\frac{I_t^c}{I^c}\right) + \left(\frac{I_t^c}{I^c}\right)\partial\Phi_t^c \tag{38}$$

$$1 = \mathbb{E}_t\left[\Lambda_{t,t+1} R_{t+1}^{kc}\right] \tag{39}$$

$$R_t^{kc} = \frac{\alpha_c p_t^c \dfrac{Y_t^c}{K_{t-1}^c} + q_t^c \lambda^c}{q_{t-1}^c} \tag{40}$$

where $\partial\Phi_t^c$ denotes the derivative of $\Phi^c(.)$ evaluated at $\frac{I_t^c}{I^c}$ and $(1-\tau^c)q_t^c$ is the shadow price for the commodity sector specific stock of capital.

Finally, we assume that a fraction $(1-\chi^c)$ of the profits is transferred abroad to foreign owners. The aggregate profit in the commodity sector is

[16] We assume that foreign stochastic discount factor is the same of their domestic counterpart. Hence, we use $\Lambda_{t,t+1}$ as the discount factor for future commodity sector's cash flows independent of its ownership.

$$\Pi_t^c = p_t^c A^c (K_{t-1}^c)^{\alpha_c} - \left[1 + \Phi^c \left(\frac{I_t^c}{I^c}\right)\right] I_t^c \tag{41}$$

In our framework, fluctuations in commodity prices induce a wealth effect in the domestic economy that is reinforced by the existence of a strong co-movement between commodity prices and the real exchange rate. Since exchange rate dynamics directly affects the agency problem faced by banks, financial conditions are relaxed (tighten) whenever the domestic economy faces higher (lower) commodity prices.

2.5 External Sector

The foreign demand for non-commodity exports of the domestic final goods is increasing with respect to the real exchange rate, e_t, and global demand or foreign income Y_t^*:

$$Y_t^{nc,x} = e_t^{\varphi} Y_t^* \tag{42}$$

where $\varphi > 0$ is the price elasticity.

The external block has its own dynamics without any feedback from domestic variables. The variables considered in the external block are: foreign output Y_t^*, the foreign interest rate R_t^*, and the commodity price index p_t^{wc}. The cyclical components of those variables are collected in vector $\widehat{\boldsymbol{X}}_t$,

$$\widehat{\boldsymbol{X}}_t = \begin{bmatrix} \widehat{Y}_t^* \\ \widehat{R}_t^* \\ \hat{p}_t^* \end{bmatrix}$$

where $\widehat{Y}_t^* = ln\frac{Y_t^*}{Y^*}$, $\widehat{R}_t^* = R_t^* - R^*$, and $\hat{p}_t^* = ln\frac{p_t^{wc}}{p^{wc}}$. We assume that $\widehat{\boldsymbol{X}}_t$ follows a first-order vector autoregressive system given by

$$\widehat{\boldsymbol{X}}_t = \boldsymbol{C}\widehat{\boldsymbol{X}}_{t-1} + \boldsymbol{B}\boldsymbol{u}_t^X \tag{43}$$

where $\boldsymbol{C}$ and $\boldsymbol{B}$ are 3×3 matrices that rule the dynamics of the vector $\widehat{\boldsymbol{X}}_t$, and $\boldsymbol{u}_t^X$ is the vector of external structural shocks. Section 3 presents further details about the data, estimation, and identification strategy of eq. (43).

2.6 Central Government

The consolidated government sector collects taxes from households and receives a fraction τ^c of the aggregate profits generated by commodity producers/exporters. These resources are then used to finance public consumption G_t and central bank operational losses, denoted by CB_t:

$$\tau^c \Pi_t^c + T_t = CB_t + G_t \tag{44}$$

Eq. (44) indicates that commodity price movements as well as central bank operational losses affect household's decisions through variations in lump-sum taxes.

The monetary authority sets the short-term nominal interest rate i_t according to a simple rule that belongs to the class of Taylor-type rules given by:

$$i_t - i = \rho_i(i_{t-1} - i) + (1 - \rho_i)\left[\omega_\pi \pi_t + \omega_y ln\left(\frac{\mathrm{GDP}_t}{\mathrm{GDP}}\right)\right] \tag{45}$$

where ρ_i measures the persistence of the policy rate and ω_π controls the degree of the policy rate response to inflation. In order to converge to a stable equilibrium, the parametrization of the above rule should satisfy the Taylor principle; i.e., $\omega_\pi > 1$.

2.7 Market Equilibrium

The non-commodity output net of adjustment, transaction, and management costs is either consumed, invested or exported.

$$Y_t^{nc} - \mathrm{REST}_t = C_t + G_t + I_t^{nc} + I_t^c + Y_t^{x,nc} \tag{46}$$

where

$$\mathrm{REST}_t = \frac{\kappa}{2}\pi_t^2 Y_t^{nc} + e_t \frac{\kappa_{D*}}{2}\left(D_t^{*,h} - \overline{D}^{*,h}\right)^2 + \frac{\kappa_S}{2}(\mathcal{S}_t - \overline{\mathcal{S}})^2$$
$$+\Phi^c(I_t^c/I^c) + \Phi^{nc}(I_t^{nc}/I^{nc}) + L_t + e_t L_t^* - \mathcal{F}_t$$

The term REST_t captures several additional costs that are present in the model economy such as investment and price adjustment costs, transaction costs for households' foreign currency deposits and equity portfolio investments. Finally, the last term in REST_t is the difference between the aggregate amount of real loans received from the banking sector and the aggregate loan services that intermediate good producers end up using to buy capital. We interpret this difference as a management cost that intermediate good producers must incur to effectively generate the optimal bundle of loans needed to produce.

The market-clearing condition for foreign currency implies that aggregate deposits denominated in foreign currency is composed by household's bank deposits and bank's foreign borrowing. Thus,

$$D_t^* = D_t^{*,h} + D_t^{*,f} \tag{47}$$

Gross Domestic Product (GDP) is defined as the aggregate value added of the total production in the non-commodity and commodity sectors but evaluated at constant prices:

$$\mathrm{GDP}_t = Y_t^{nc} - eM_t + p^c Y_t^c \tag{48}$$

where p^c and e are the steady-state levels for the commodity price index and the real exchange rate, respectively. Therefore, GDP_t captures only real output movements and is not affected by valuation effects.

The aggregate net foreign asset position of the economy NFAP_t, is equal to FX official reserves net of aggregate foreign liabilities in the baking system (i.e., $F_t - D_t^{*,f}$), and evolves as a function of financial income of net foreign assets from the previous period plus the difference between the trade balance and the fraction of aggregate profits in the commodity sector transferred abroad,

$$e_t[\mathrm{NFAP}_t - R_t^* \mathrm{NFAP}_{t-1}] = Y_t^{x,nc} + p_t^c Y_t^c - e_t M_t - (1-\tau^c)(1-\chi^c)\Pi_t^c \tag{49}$$

Finally, the aggregation for the banking system is straightforward since optimal banks' decisions do not depend on bank-specific factors. In Appendix C.1, we show that aggregate net worth for banks evolves according to:

$$N_t \quad = (\sigma + \xi)(R_t^l L_{t-1} + R_t^{l*} e_t L_{t-1}^* + R_t^b B_{t-1}) - \sigma R_t D_{t-1} - \sigma e_t R_t^* D_{t-1}^* \tag{50}$$

3. Parametrization Strategy

We discipline the model to replicate some relevant unconditional and conditional moments for the Peruvian economy. We calibrate a subset of the parameters to be consistent with some steady state targets associated with historical averages. Additionally, we follow Schmitt-Grohé and Uribe (2018) to estimate another subset of parameters by using a limited information method based on an impulse response matching function estimator. For this purpose, we estimate an SVAR with two recursive blocks for a small open economy. Then, we estimate some parameters of our macroeconomic model by minimizing the distance between the structural impulse responses implied by the macroeconomic model and the corresponding empirical impulse responses implied by the SVAR model. Let $\varXi$ be the subset of parameters to be estimated by matching the impulse responses to external shocks, $\mathcal{M}^{\text{data}}$ the corresponding empirical impulse responses from the SVAR model, and $\mathcal{M}^{\text{model}}$ the theoretical counterpart of $\mathcal{M}^{\text{data}}$. Then we set $\varXi$ to be the solution to the following problem

$$\varXi^* = \text{arg}\min_{\varXi} \sum_{i=1}^{k} \varrho_i^{-1} \times \left[\mathcal{M}_i^{\text{model}}(\varXi) - \mathcal{M}_i^{\text{data}}\right]^2 \tag{51}$$

where ϱ_i is a scalar containing the width of the 68% confidence interval associated with the ith variable in $\mathcal{M}^{\text{data}}$. This scalar penalizes the elements of the estimated impulse response functions associated with large error intervals.

Empirical VAR Specification. We consider an SVAR model with two recursive blocks similar to Canova (2005), Cushman and Zha (1997), and Zha (1999). Let $\boldsymbol{X}_t$ denote the vector of foreign variables and $\boldsymbol{D}_t$ the vector of domestic variables. In the baseline specification, each block is composed by the following variables:

$$\boldsymbol{X}_t = \begin{bmatrix} Y_t^* \\ R_t^* \\ p_t^{wc} \end{bmatrix}, \boldsymbol{D}_t = \begin{bmatrix} tb_t \\ GDP_t \\ C_t \\ I_t \\ L_t \\ eL_t^* \\ e_t \end{bmatrix}$$

The external variables Y_t^*, R_t^*, and p_t^{wc} denote the real GDP index for the G-20 group of countries, the Baa U.S corporate spread, and a metal export price index relevant for the Peruvian economy. The domestic variables GDP_t, C_t, I_t, L_t, and $e_tL_t^*$ denote real indexes for Peru's GDP, consumption, investment, and real bank lending in domestic currency as well as in foreign currency respectively, while $e_t$denotes the bilateral real exchange rate and tb_t the trade balance-to-GDP ratio. Following Canova (2005), the baseline specification considers $\boldsymbol{X}_t$ as an exogenous block, with no feedback dynamics from the domestic block, $\boldsymbol{D}_t$, at any point in time. Therefore, like much of the related literature, the main identification assumption is that an emerging small open economy such as Peru takes as given world prices and quantities. The baseline specification assumes that all variables are expressed in log-levels. The only variables expressed in percentage terms are R_t^* and tb_t. Therefore, we consider an SVAR in levels with zero restrictions between blocks and a linear or quadratic time trend to capture the SOE assumption of the Peruvian economy, as well as to control for time trends, respectively. It is important to mention that shocks within each block are identified recursively with zero contemporaneous restrictions.

Formally, consider the following restricted block VAR model with deterministic trend:

$$\begin{bmatrix} \boldsymbol{X}_t \\ \boldsymbol{D}_t \end{bmatrix} = \begin{bmatrix} \Phi_X \\ \Phi_D \end{bmatrix} G(t) + \begin{bmatrix} \Phi_{XX}^1(L) & 0 \\ \Phi_{DX}^1(L) & \Phi_{DD}^1(L) \end{bmatrix} \begin{bmatrix} \boldsymbol{X}_{t-1} \\ \boldsymbol{D}_{t-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{v}_t^X \\ \boldsymbol{v}_t^D \end{bmatrix}$$

where $G(t)$ measures a deterministic time trend.[17] Φ_X, Φ_D are vectors of ones, $\boldsymbol{v}_t^X \sim \mathcal{N}(0, \Sigma_{\boldsymbol{v}^F})$ and $\boldsymbol{v}_t^D \sim \mathcal{N}(0, \Sigma_{\boldsymbol{v}^D})$. Hence, the underlying SVAR model is

$$\begin{bmatrix} \Theta_{XX}^0 & 0 \\ \Theta_{DX}^0 & \Theta_{DD}^0 \end{bmatrix} \begin{bmatrix} \boldsymbol{X}_t \\ \boldsymbol{D}_t \end{bmatrix} = \begin{bmatrix} \Theta_X \\ \Theta_D \end{bmatrix} G(t) + \begin{bmatrix} \Theta_{XX}^1(L) & 0 \\ \Theta_{DX}^1(L) & \Theta_{DD}^1(L) \end{bmatrix} \begin{bmatrix} \boldsymbol{X}_{t-1} \\ \boldsymbol{D}_{t-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{u}_t^X \\ \boldsymbol{u}_t^D \end{bmatrix}$$

We use quarterly data, covering from 2002Q1 to 2017Q2 for the domestic block and from 1980Q1 to 2017Q2 for the foreign block. Following Fernández et al. (2017), we first estimate the foreign block separately and impose the corresponding estimated parameters in the estimation of the domestic block.

Calibrated Parameters. The elasticity of intertemporal substitution for household preferences is equal to $1/2$ (i.e., $\gamma = 2$). Consistent with Céspedes and Rendón (2012), households' preferences have a Frisch elasticity of the labor supply equal to $1/3$ (i.e., $\zeta = 3$). With respect to the production sector, the elasticity of substitution among intermediate goods is set at 6 and the capital depreciation rate is set at 10 percent annually for both sectors. We also assume that foreign agents have the ownership of commodity firms (i.e., $\chi^c = 0$) but there is a commodity

[17] Like the SVAR model, the DSGE model considers deterministic time trends that are removed before the matching procedure.

profit tax of 60 percent, which is in line with García-Cicco et al. (2017). The parameters controlling the conventional monetary policy response (ρ_i; ω_π; ω_y) are parametrized using previous work (see for example Castillo et al., 2009, and Winkelried, 2013). Following ABK, we fix the bank's survival rate, σ, at 0.945. Moreover, in our baseline analysis we ignore efficiency costs of the FX operations, i.e., $\tau^{fx} = 0$.

TABLE 2. *Raw parametrization*

Description	Parameter	Value
Elasticity of Intertemporal Substitution	γ	2.00
Inverse Frisch Elasticity	ζ	3.00
Elasticity of Substitution of Goods	η	6.00
Undepreciated NC Capital Rate	λ^{nc}	0.975
Undepreciated C Capital Rate	λ^{c}	0.975
Domestic Ownership on Commodity Firms	χ^{c}	0.00
Tax on Commodity Sector Profit	τ^{c}	0.60
Banker's Survival Rate	σ	0.94
MP Rate Smoothing	ρ_i	0.70
MP Rate response to Inflation	ω_π	1.50
MP Rate response to Output Gap	ω_y	0.125
FXI efficiency cost	τ^{fx}	0.00

Although these parameters are calibrated to be consistent with previous literature for the Peruvian economy, it is worth mentioning that their values are valid for any typical emerging market economy. Table 2 summarizes the parametrization described above.

Additionally, we parametrize (ϖ^*, ϖ^b, ξ, θ, $\varkappa$, δ^f) to be consistent with the following steady-state financial targets for Peru: an annual domestic currency lending rate of 6 percent, an annual foreign currency lending rate of 4 percent, an annual return of 4 percent for central bank bonds, a domestic currency leverage of 3.00, dollar deposits to total assets ratio of 60 percent, and a credit dollarization rate of 50 percent. See Figure 1 and Figure 13 for the evolution of the banking system variables in Peru from 2002 to 2018.

Similarly, the vector of parameters (A^e, ζ_0, Y^*, p^{wc}, A^{nc}) is calibrated to attain 8 percent of annual non-commodity capital return, 0.33 of worked hours and steady-state levels for the real exchange rate, the commodity price index, and GDP normalized at 1. Furthermore, the vector

($\overline{D}^{*,h}$, $\overline{S}$, α^c, A^c, α^k, α^m, B^{fx}) is parametrized to target some empirical ratios, such as consumption to GDP, commodity to non-commodity investment or FX reserves to GDP, among others. Table 11 in Appendix A summarizes our parametrization strategy based on the steady-state targets described above.

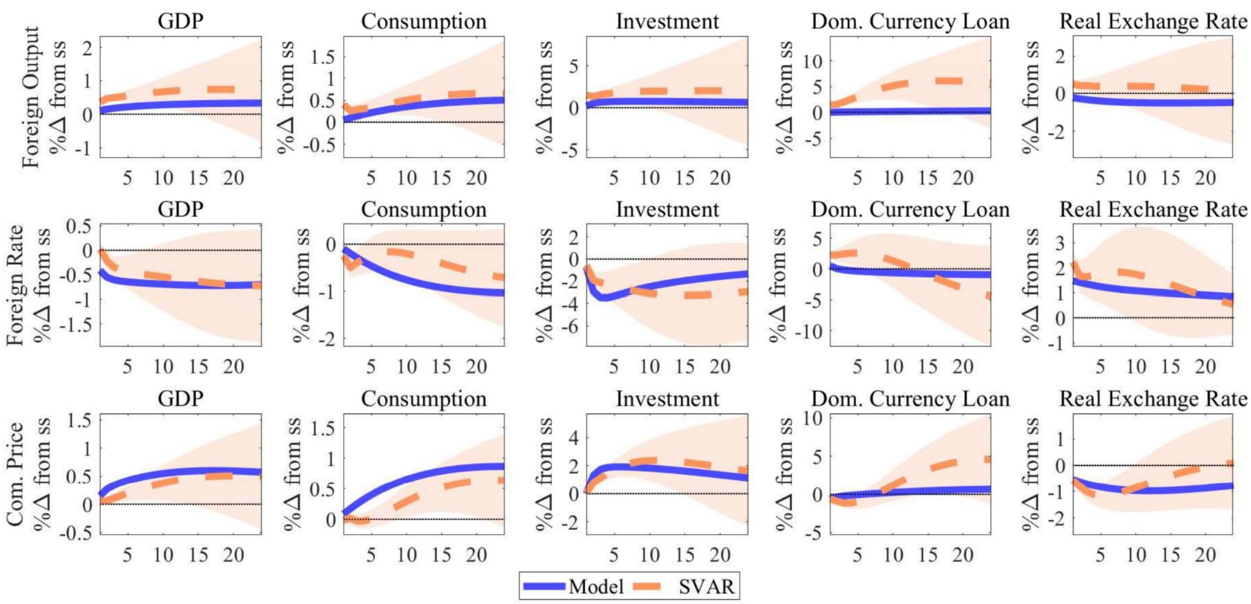
TABLE 3. *IRF matching estimation*

Description	Parameter	Value
Non-Commodity Capital Adjustment Cost	κ_{Inc}	0.05
Commodity Capital Adjustment Cost	κ_{Ic}	0.15
FX Intervention response to RER	υ_e	9.71
Non-Commodity Exports Price Elasticity	φ	0.09
Household FC Deposit Adjustment Cost	κ_{D*}	3.00
Houosehold Capital Adjustment Cost	κ_S	0.02
Household Habit Formation	$\mathcal{H}$	0.90

Impulse Response Matching. The rest of the parameter set is estimated to match impulses responses to external shocks between the SVAR model and the DSGE model. We use the responses of GDP, consumption, investment, domestic currency-denominated loans (DC loans), and the real exchange rate for the first 24 quarters to perform the matching estimation.

Our estimation results are summarized in Table 3. Figure 3 compares the corresponding impulse-responses. Our empirical model indicates that a foreign interest rate shock causes a real exchange rate depreciation and a contraction on aggregate credit and output. On the other hand, the global demand and the commodity price shocks are expansionary in terms of domestic output, investment, and total credit. These empirical responses are very closely followed by the theoretical responses of our DSGE model.

FIGURE 3. *Impulse response matching estimation*



Note. Solid blue (orange-dashed) lines show point estimates of impulse response of the DSGE model (SVAR model); and 68% confidence bands associated with the SVAR's impulse response are depicted with orange-shaded areas.

Table 4 reports that second moments such as the relative standard deviation with respect to GDP and first order autocorrelation of the GDP, consumption, investment, and real exchange rate are very close to their empirical counterparts. All in all, our baseline parametrized model fits quite well the aggregate business cycles observed in data for a typical small open and commodity exporter economy with financial dollarization.[18]

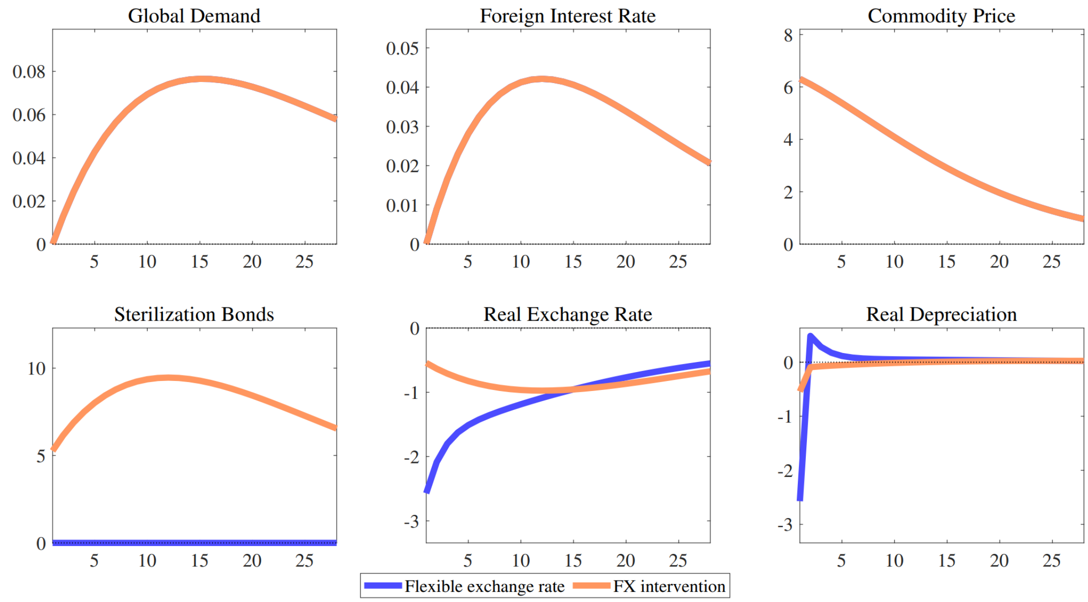
[18] Excluding the parameters associated with financial dollarization (δ^f, κ_{D*}, and $\overline{D}^{*,h}$) as well as the endogenous response of FX operations (υ_e), our baseline calibration is consistent to the any typical emerging market economy. Moreover, to gain robustness for our baseline analysis, in we explore more general assumptions about the financial sector in our baseline model and find that the key results are still valid.

TABLE 4. *Data versus Model Second Moments*

	Gross Dom. Product	Consumption	Investment	Real Exchange Rate
RELATIVE STANDARD DEVIATION W.R.T. GDP				
Data	1.00	0.98	5.10	2.24
Model	1.00	0.83	5.86	2.41
AUTOCORRELATION (1ST ORDER)				
Data	0.91	0.87	0.91	0.93
Model	0.85	0.96	0.85	0.71

Remarkably, Figure 3 indicates that, in response to a commodity price shock, the empirical impulse responses from the SVAR model are consistent with hump-shaped dynamics for the real exchange rate. In our model, FX interventions modify the response of exchange rate expectations as well as the relative costs and returns of borrowing and lending in foreign and domestic currency when compared to the flexible exchange rate regime. These modifications play a key role in generating hump-shaped movements for the real exchange rate in our model. In Figure 4, we compare the real exchange rate response to a commodity price shock under both exchange rate regimes. The foreign block in our model is calibrated as in the estimated SVAR. Therefore, the commodity price shock generates a positive co-movement between the external variables in the SVAR as well as in the structural model. The baseline parametrization of our model indicates that under exchange rate flexibility, a commodity boom generates a real exchange rate appreciation that only occurs at impact, undershooting its long-run equilibrium level. After impact, the exchange rate is below its steady state and depreciates in every subsequent period. Hence, under exchange rate flexibility agents expect an exchange rate depreciation. On the contrary, after a commodity boom, the FX intervention policy generates consecutive exchange rate appreciations for about 10 quarters, implying that the same shock induces different expectation dynamics between both exchange rate regimes.

FIGURE 4. *Commodity price shock, FX intervention, and real exchange rate*



Note. The solid black line plots the response of the baseline under a sterilized foreign exchange regime while the blue dash-line plots the aggregate variables response of the model under a flexible exchange rate regime.

4. Numerical Experiments

In this section, we perform several simulations designed to analyze how FX intervention operations affect the response of the model economy to external shocks. Specifically, we focus on the transmission of a sudden increase in the foreign interest rate and a global commodity boom.[19]

The foreign block in our baseline model is calibrated as in the estimated SVAR, but we begin by analyzing the responses of aggregate variables to external shocks under two different exchange rate regimes: exchange rate flexibility (FER) vs. an FX intervention policy (FXI) regime. Under the FX intervention regime, the central bank "leans against the wind" with respect to real exchange rate fluctuations by implementing eq. (16), but its interest rate rule is also active. Next, we simulate an exogenous, sufficiently large and permanent unanticipated accumulation (purchase) of FX reserves and study the transmission mechanisms for this shock. Finally, we conduct a policy evaluation exercise by computing the welfare gains/costs of different policy regimes relative to our baseline regime and analyze the optimal FX intervention rule.

[19] In Appendix B.3, we also show the responses to an increase in global GDP.

Recent empirical literature on FX policy (see Fratzscher et al., 2019) uses distinct criteria to measure the effectiveness of the FX intervention policy. In this line, our numerical experiments can be seen as designed to evaluate these criteria. For instance, our impulse-response analysis can be associated with the event criterion which tests whether the exchange rate moves in the intended direction during the intervention episode (e.g., if the central bank buys foreign currency, the real exchange rate should depreciate). At the same time, in line with the exchange rate dynamics smoothing criterion, we evaluate whether FX interventions limit the real exchange rate volatility (see Table 5). Although this literature has studied the effectiveness of FX intervention in terms of the exchange rate volatility, we extend its usage to analyze the FX policy effectiveness over a broader set of macroeconomic variables. Additionally, we use the general equilibrium framework of our model economy to explore the effectiveness of the FX intervention policy in terms of welfare relative to exchange rate flexibility.

TABLE 5. *Aggregate volatility conditional on external shocks*

	FXI	FER	Δ%
RER	2.07	5.48	-62
Inflation	0.34	0.74	-53
UIP Dev.	0.30	1.36	-78
GDP	0.86	1.91	-55
Investment	5.03	10.31	-51
Total Credit	1.04	3.88	-73
Currency Mismatch	1.79	4.35	-59
Consumption	0.71	0.94	-24
Labor	0.40	0.51	-23

Note. Standard deviations for major aggregate variables. FXI and FER denote Foreign Exchange Intervention and Flexible Exchange Rate policy regime respectively. The computation consider the external block as the only source of aggregate volatility and it is based on 2500 replications of 120 periods simulated trajectories.

Our results suggest that when financial constraints are binding (i.e., limits to arbitrage emerge for banks and households leading to endogenous deviations from UIP conditions), FX intervention operations play the role of an external shock absorber: conditional on external shocks, macroeconomic volatility is significantly reduced under the FX intervention policy relative to the flexible exchange rate regime. In particular, the volatility of the real exchange rate (RER) is reduced by 68 percent, while the corresponding volatility of total credit declines by 82 percent (see Table 5). Simultaneously, the volatility of output, investment, and consumption falls by around 70, 65, and 7 percent, respectively. Therefore, according to the *volatility smoothing criterion*, FX interventions are significantly effective in stabilizing domestic macroeconomic volatility conditional on external shocks.[20]

In the following numerical experiments, we discuss the mechanisms through which FX intervention policy stabilize domestic macroeconomic variables responses in the presence of external shocks.

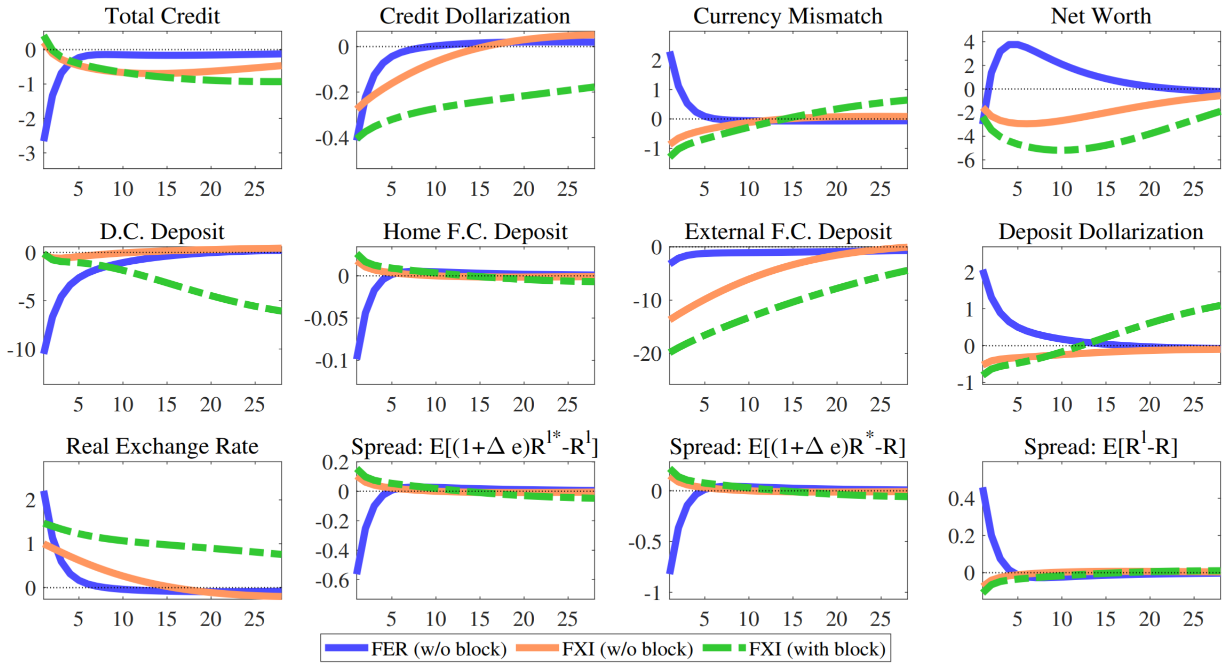
4.1 Responses to External Shocks

4.1.1 Foreign Interest Rate Shock

Figure 5 and Figure 6 show responses to an unexpected increase of 20 basis points in the foreign interest rate of financial and macroeconomic variables, respectively. The green-dashed line and the orange line report the responses under the FX intervention policy (FXI) for two cases: i) Considering all the co-movements among the external variables as estimated in the SVAR model and ii) The case where those co-movements are muted in order to isolate the effect of each external shock. On the other hand, the solid blue line represents the economy under exchange rate flexibility (FER) ($\upsilon_e = 0$, in (16)) with muted co-movements in the foreign block. We first describe the transmission mechanism under exchange rate flexibility then compare it to the responses under the FXI regime under both scenarios (complete and muted co-movements in the foreign block).

[20] Even though the FX intervention rule eq. (16) implies a linear response to any deviation of the real exchange rate from its steady-state, the reduction of macroeconomic volatility is not a necessary result. Below, we explore cases where the FX intervention regime modelled as in eq. (16) does not succeed in terms of the *volatility smoothing and welfare criterion*.

FIGURE 5. *Responses of financial variables to a foreign interest rate shock*



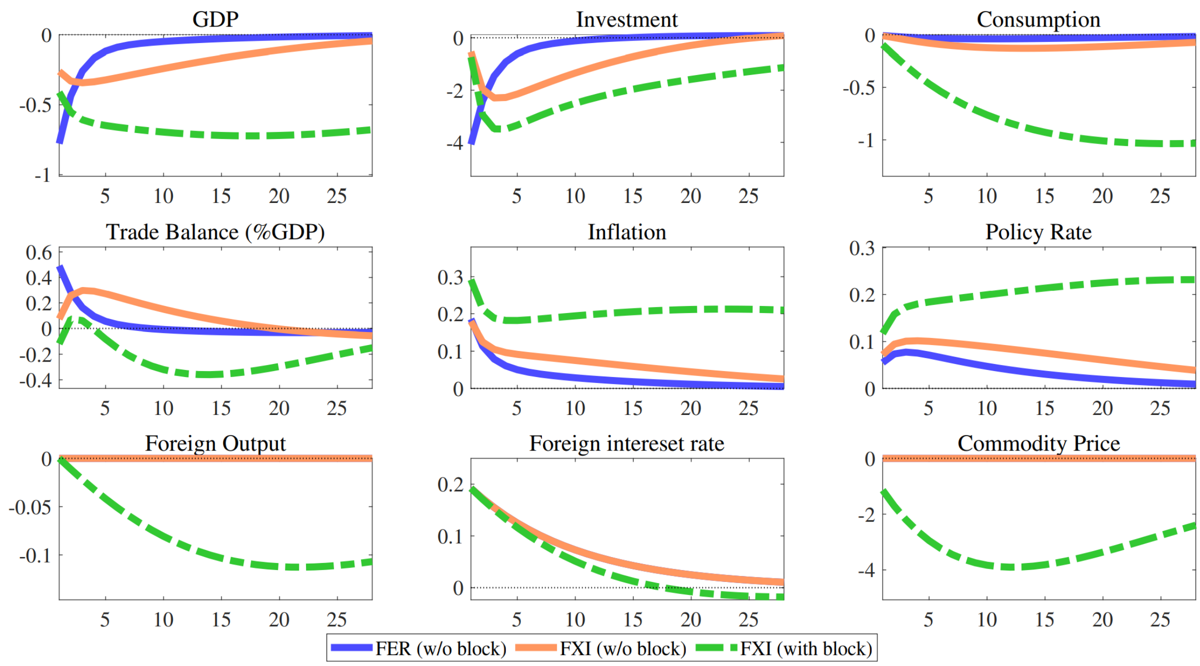
Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate/spread variable is displayed in percentage-point deviations from its steady-state.

FER with muted co-movements in the foreign block. Initially, the real exchange rate depreciates by 2.1 percent and the economy experiences a contractionary financial effect. Since banks are exposed to currency mismatches in their balance sheets, the real exchange depreciation negatively affects banks' net worth and total credit, and ultimately generates a recession. Net worth declines at impact but shows a fast recovery and then stabilizes around zero.

Although the real exchange rate depreciates immediately after the shock, agents expect an exchange rate appreciation (see the dynamics of real exchange rate in Figure 5). The expected exchange rate appreciation modifies the relative costs and returns of lending and borrowing in foreign currency with respect to domestic currency, thereby changing the composition of banks' balance sheets. Banks realize that lending in foreign currency becomes less profitable than lending in domestic currency, and, consequently, bank's credit dollarization falls (an impact of -0.4 percentage points right after the shock). Similarly, borrowing in foreign currency is cheaper than in domestic currency and banks' deposit dollarization rises: the fall in domestic currency deposits is higher than the reduction of foreign currency deposits. However, the aggregate level of loans and deposits decline due to banks' net worth contraction together with the aggregate demand recession.

Hence, under a flexible exchange rate regime, the exchange rate depreciation induced by a higher foreign interest rate, boosts the size of the currency mismatch, thereby reducing the intermediation capacity of banks, so that lending in both currencies declines by around 2.7 percent.

FIGURE 6. *Responses of macroeconomic variables to a foreign interest rate shock*



Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate/spread variable is displayed in percentage-point deviations from its steady-state.

Financial conditions are reflected in interest rate spreads and macroeconomic variables. In particular, right after the foreign interest rate increases, the reduction of the bank's lending capacity is mirrored in a higher expected interest rate spread of domestic currency lending relative to domestic currency borrowing (by 0.4 annual percentage points). Therefore, investment falls by 4 percent, leading to a prolonged recession, with GDP falling by 0.7 percent (see Figure 6). Finally, exchange rate depreciation raises inflation by 0.2 percent on impact, since the marginal cost of intermediate good producers depends on an imported input. The increase in inflation leads to a higher interest rate due to the standard Taylor rule mechanism.

FXI with muted co-movements in the foreign block. When the central bank responds to a foreign interest rate shock implementing FX intervention policy, together with its standard monetary policy rule, both financial and macroeconomic variables are stabilized relative to the flexible exchange rate regime. The effect of FX interventions on the transmission mechanism of an external shock operates through two main channels: the exchange rate smoothing channel and the balance sheet substitution channel.

Exchange Rate Smoothing Channel. When the incentive constraint binds, FX interventions modify the net asset foreign position of the aggregate economy, as well as the interest rate spread between foreign borrowing and domestic deposits that firms, households and banks face. In particular, the central bank responds to an increase in the foreign interest rate by selling official FX reserves. Therefore, exchange rate dynamics change relative to the flexible exchange rate regime. At impact, the real exchange rate depreciates by 1.0 percent under the FX intervention regime, instead of 2.1 percent under the flexible exchange rate regime. After the impact, FX interventions successfully stabilize future real exchange rate appreciations.

As a result, banks' net worth declines less at impact under the FX intervention regime (around 1.5 percent instead of 2.8 percent under the flexible exchange rate regime, see Figure 5). The smoother pattern for the real exchange rate modifies the cost of borrowing in foreign currency relative to domestic currency deposits. In particular, under the FX intervention regime, the expected interest rate spread of domestic-currency borrowing over domestic-currency deposits raises around 0.2 percentage points instead of falling in 0.8 percentage points under the free-floating exchange rate. Hence, contrary to the free-floating regime, deposit dollarization declines by 0.5 percentage points at impact.

Moreover, external foreign currency deposits decline more than under the free-floating exchange rate regime (about 15 percent rather than 3 percent). This model's prediction is not obvious and is the result of distinct responses on the home deposits market. On the one hand, the demand for total foreign-currency deposits relative to domestic-currency liabilities falls due to the expected real depreciation. On the one hand, the supply of home foreign currency deposits increases because the expected real exchange depreciation induces households to hold more foreign currency-denominated savings. Therefore, the only mechanism for banks to reduce their liability's dollarization is by borrowing less from the international markets.

Similarly, the expected interest rate spread of foreign-currency loans over domestic currency loans is more stable, implying that credit dollarization falls but not as much as under exchange rate flexibility. Although the FX operations change the direction of the response for this spread, the credit dollarization rate continuous falling as the expected real exchange depreciation contracts the relative demand of foreign-currency loans.

Balance Sheet Substitution Channel. This channel is associated with central bank sterilization operations to keep domestic liquidity constant after FX sales. The central bank buys bonds that are in banks' balance sheets, ultimately affecting their size and composition. Consequently, this operation releases bank funds, which are used to lend in both currencies. In this regard, FX interventions are similar to credit policies in the non-conventional monetary policy literature for closed economies.

Quantitatively, our results suggest that the sterilization leg of FX sales implies that central bank bonds in banks' balance sheets decline by around 9.7 percent at impact ($= 9.7 \times 1.0$). As a result, lending denominated in both currencies decline less than under exchange rate flexibility. In particular, at the trough of the recession, total loans fall by 0.7 percent when FX interventions are used, instead of declining by 2.7 percent under free floating.

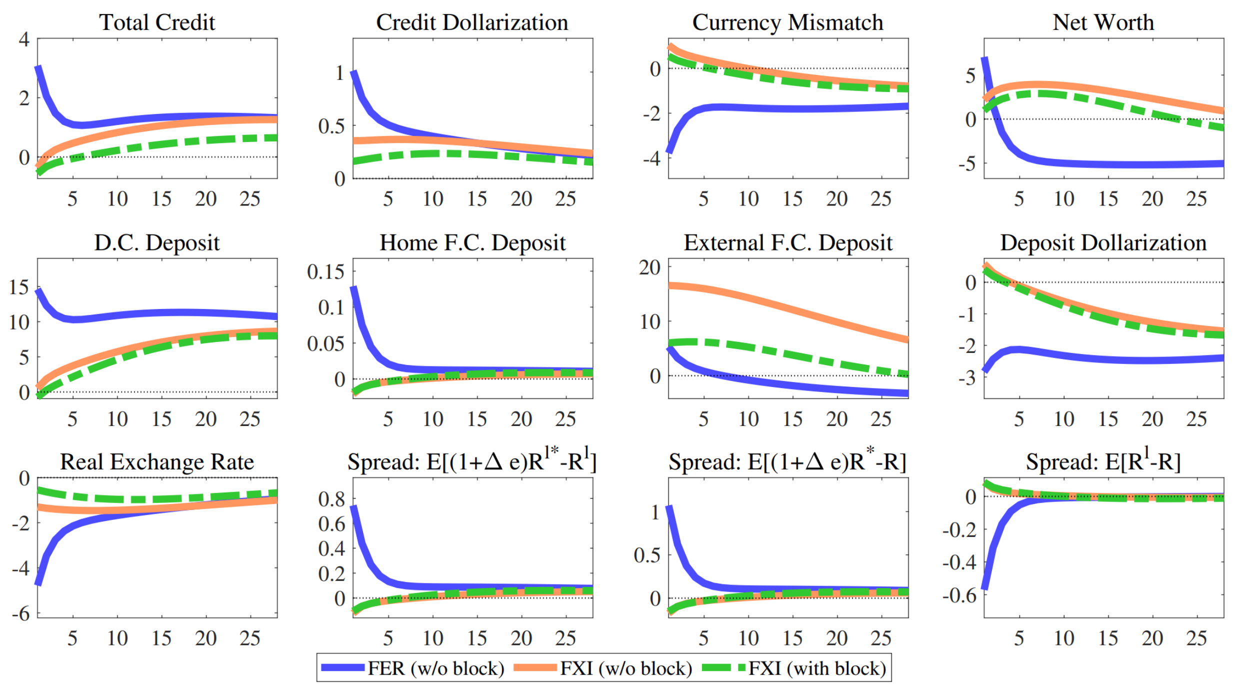
FXI considering co-movements in the foreign block. Co-movements in the foreign block add further contractionary channels through which an increase in foreign interest rate affects the domestic economy. In particular, the foreign interest rate correlates negatively with both global demand and commodity prices, then a rise in it generates a larger contraction in domestic GDP via a fall in the trade balance (see Figure 6). Moreover, distinct from the case with muted external co-movements, the dynamic interaction with the foreign block generates a highly persistent recession, which is consistent with our empirical analysis.

4.1.2 Commodity Price Shock

EMEs face volatile commodity prices that shape capital flows and domestic financial conditions. In this section, we simulate a persistent increase in commodity prices and compare the transmission mechanism of this shock under exchange rate flexibility and the FX intervention policy. Figure 7 shows the responses of financial variables, while Figure 8 presents the response of key macroeconomic variables. The blue-solid line corresponds to the flexible exchange rate regime (with muted external block co-movements), while the green-dotted and orange-solid lines represent the FX intervention regime in the case that considers external variables co-movements and in the case without them, respectively.

FER with muted co-movements in the foreign block. Under exchange rate flexibility, a persistent increase in commodity prices raises exports and a large fraction of the revenues from commodity exports remains in the economy, leading to a persistent exchange rate appreciation of around 4.5 percent at impact (see Figure 7). The commodity sector experiences a prolonged economic boom that spreads to the rest of the economy through a significant wealth effect and a higher demand for investment goods.

FIGURE 7. *Responses of financial variables to a commodity price shock*

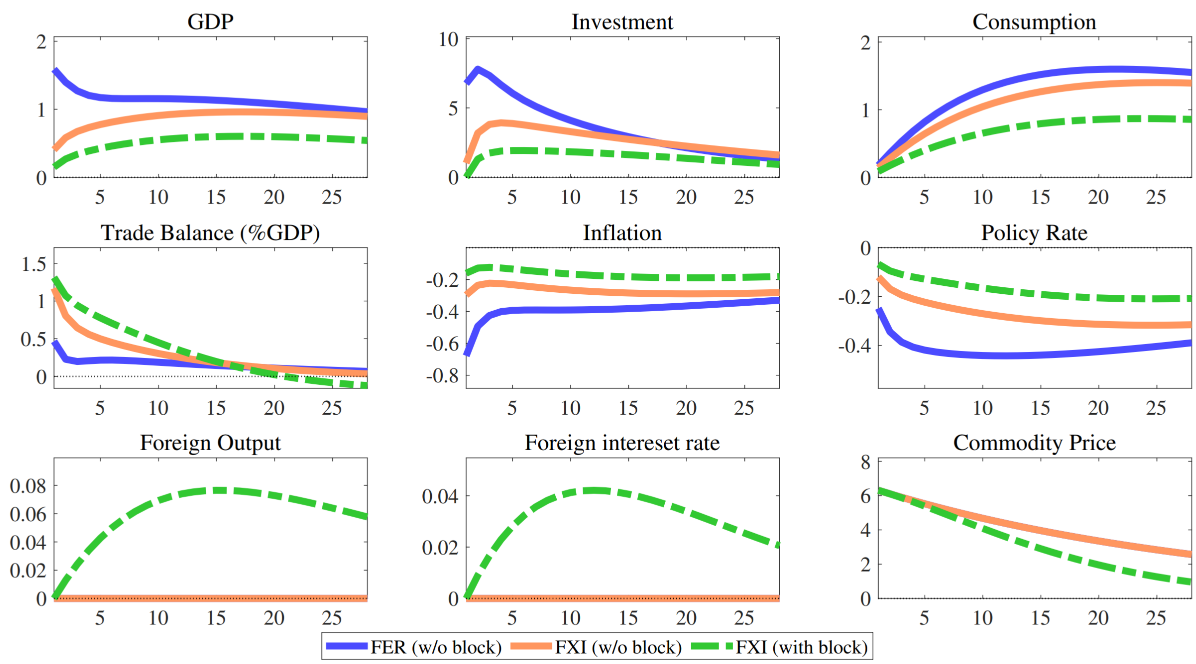


Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate/spread variable is displayed in percentage-point deviations from its steady-state.

The exchange rate appreciation relaxes the agency constraint that banks face via a 6 percent increase in net worth, together with a significant currency mismatch reduction of 4.0 percentage point right after the shock occurs. The latter is an expansionary financial effect induced by the real exchange rate appreciation. Hence, lending in both currencies rises by around 3.0 percent at impact. Under exchange rate flexibility, agents expect a real exchange rate depreciation, implying that banks realize that borrowing in foreign currency is more expensive than in domestic currency, while lending in foreign currency is more profitable than in domestic currency. The change in the composition of banks' balance sheets is consistent with a 1.0 p.p. increase in credit dollarization and a reduction of 3 percentage points in deposit dollarization at impact.

The commodity boom, together with the consequent expansionary financial conditions (i.e., credit boom), modify the dynamics of interest rate spreads and real macroeconomic variables. Specifically, the expected interest rate spread of domestic-currency lending relative to domestic-currency deposits falls around 0.6 percentage points (see Figure 7), while the expected interest rate spread of foreign borrowing with respect to domestic-currency deposits rises by 1 percentage points. Investment and consumption increase persistently by around 8.0 percent and 1.6 percent at the peak of their responses, respectively (see Figure 8). The commodity boom under a flexible exchange rate regime induces a period of persistent economic expansion, with GDP increasing by 1.6 percent at impact.

FIGURE 8. *Responses of macroeconomic variables to a commodity price shock*



Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate/spread variable is displayed in percentage-point deviations from its steady-state.

FXI with muted co-movements in the foreign block. When FX intervention policy is used, the central bank accumulates FX reserves and allocates central bank riskless bonds to the banking system as a response to higher commodity prices and the appreciatory pressures on the real exchange rate. Given the binding agency problem, accumulating FX reserves significantly reduce exchange rate appreciation, thereby limiting the expansion of bank credit and the consequent expansion in macroeconomic aggregates such as consumption, investment, and GDP. As mentioned before, FX interventions operate through the exchange rate smoothing channel and the balance sheet substitution channel.

Exchange Rate Smoothing Channel. The central bank responds to a commodity price shock by buying FX reserves, thereby modifying the net foreign asset position of the economy. As a result, exchange rate dynamics change relative to the flexible exchange rate regime. At impact, the real exchange rate appreciates by 1.6 percent instead of 4.5 percent (see Figure 7). Consequently, at impact banks' net worth increases less than under free floating (2 percent instead of 6 percent). Moreover, the smoother pattern of real exchange rate modifies the costs and returns of foreign-currency borrowing and lending. When the central bank implements FX intervention, banks increase foreign borrowing together with domestic deposits, implying higher deposit dollarization relative to the flexible exchange rate regime (see the responses of spreads in Figure 7). Nonetheless, the expected appreciation signals households to supply less foreign currency deposits, and, consequently, the unique adjustment for banks to increase deposit dollarization is to borrow more from external agents: external foreign currency deposits increase by 16 percent instead of 5 percent under the FER regime.

Likewise, the expected real exchange rate appreciation under FX intervention signals banks that foreign currency lending is more profitable than lending in domestic currency. However, the same expectation induces intermediate good producers to demand more foreign currency loans relative to domestic currency loans. Therefore, credit dollarization increases, but less than under exchange rate flexibility.

Balance Sheet Substitution Channel. When the central bank responds to a commodity price shock by building FX reserves, a sterilization operation is implemented simultaneously; i.e., central bank bonds are sold to maintain the domestic liquidity constant. As a result, the composition and size of banks' balance sheets change, ultimately generating a crowding-out effect that limits lending resources. In particular, banks allocate their increased available funds to central bank bonds instead of lending. Accordingly, lending in both currencies increases by less than under exchange rate flexibility. The muted response of aggregate credit under the FX intervention regime is reflected in the response of interest rate spreads. Figure 7 shows that the interest rate spread of domestic currency lending over domestic currency deposits rises around 0.1 p.p. when the central bank responds by building FX reserves instead of falling 0.6 percentage points under exchange rate flexibility.

FXI considering co-movements in the foreign block. In contrast with the foreign interest rate shock, the commodity price shock is consistent with co-movements in global demand and foreign interest rates that generate opposite effects in the domestic economy. For instance, according to the estimated SVAR, an increase in commodity prices is associated with external block dynamics that show a higher global demand together with higher foreign interest rates. The latter generates a contractionary effect on domestic economic activity, while the former produces an expansionary effect on the domestic economy, indicating that the net effect is not so obvious in the case of a commodity price shock as it is in the case of a foreign interest rate shock. Figure 8 suggests that, in our baseline model, an increase in commodity prices induces smoother responses for domestic real GDP, investment, consumption and inflation under the FXI regime. This result indicates that the contractionary effects associated with higher foreign interest rates during a commodity boom dominate the expansionary effect of a higher global demand. Recall that, at impact, the foreign interest rate operates through the financial sector of the model.

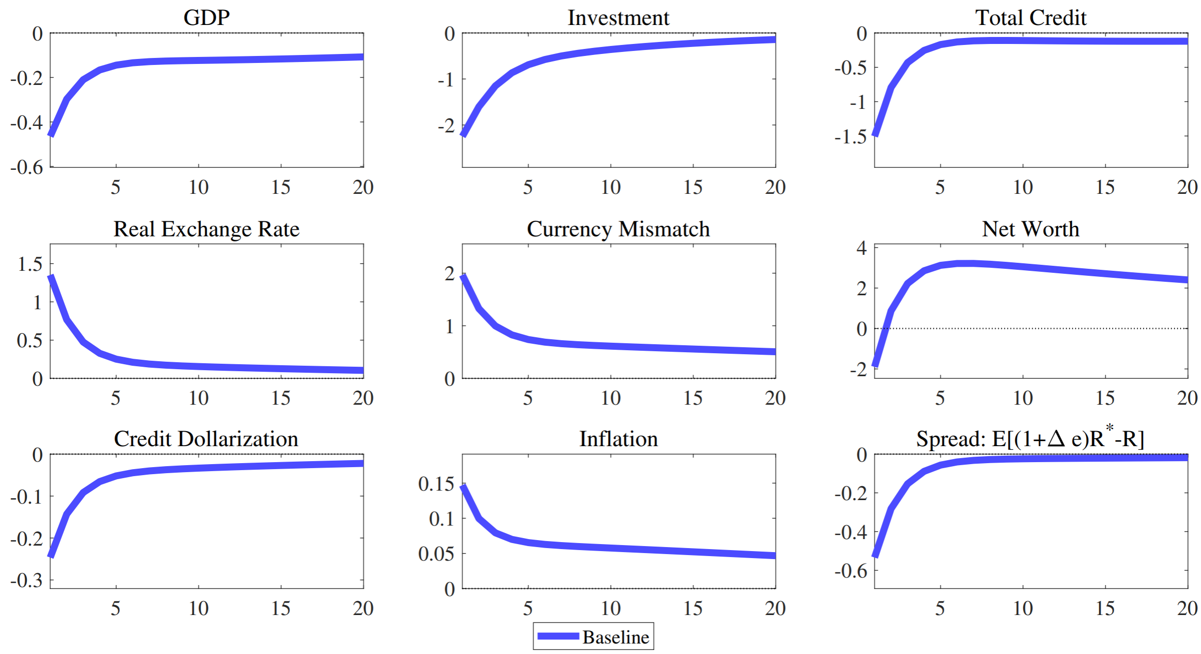
4.2 The Transmission of a Permanent Buildup of FX Reserves

In this section, we analyze the impact of an exogenous FX intervention shock to obtain more insights about the transmission mechanism. We assume the FX intervention rule is given by the following exogenous autoregressive process:

$$lnB_t - lnB = \rho_B(lnB_{t-1} - lnB) + u_t^B, \text{with } \rho_B \approx 1 \quad (52)$$

where u_t^B is interpreted as an unanticipated central bank purchase of FX reserves. Under the above process, an exogenous buildup of FX reserves has permanent effects on central bank bonds in the hands of the banking system. Figure 9 shows responses to a very persistent unanticipated purchase of FX reserves together with the corresponding sterilization operation (i.e., selling of central bank bonds to the banking system). The buildup of FX reserves induces an initial real exchange rate depreciation of around 3.5 percent that raises inflation and the monetary policy rate as well. The trade channel triggers a corresponding trade balance surplus. The balance sheet substitution channel is such that the sterilization operation modifies the asset composition of banks' balance sheet to less lending and more central bank bonds. Finally, the purchase of FX reserves by the central bank induces a financial channel as well. The real exchange rate depreciation reduces banks' net worth and raises currency mismatch at the bank level.

FIGURE 9. *Response to a persistent purchase of FX reserves*

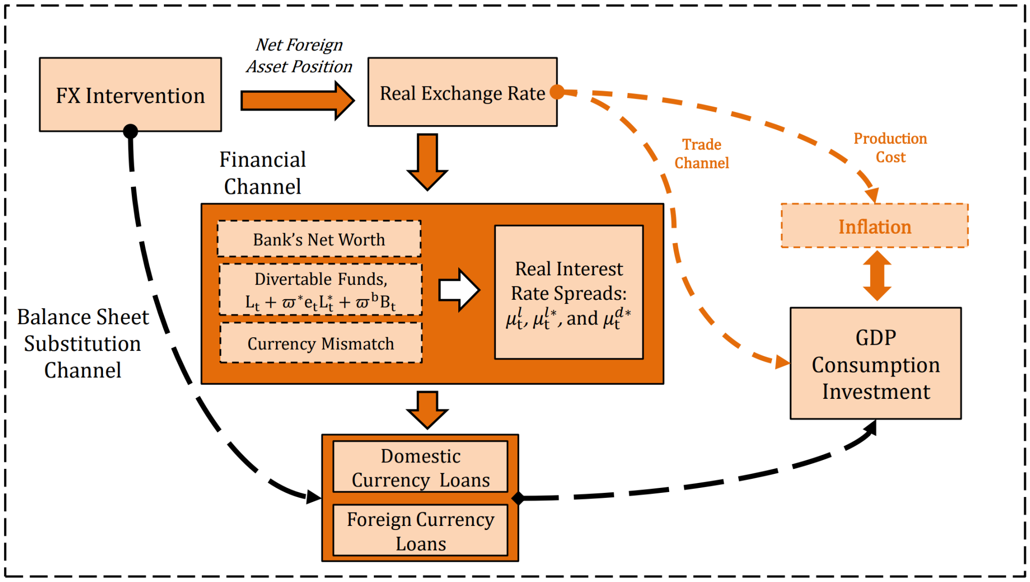


Note: The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate/spread variable is displayed in percentage-point deviations from its steady-state.

Consequently, domestic financial conditions worsen, which is reflected in higher interest rate spreads and lower aggregate credit. The real exchange rate dynamics is such that agents expect an appreciation right after the shock occurs. Therefore, deposit dollarization increases while credit dollarization falls. The financial and the balance sheet substitution channels outweigh the trade channel. As a result, the persistent and exogenous buildup of FX reserves pushes the economy to a credit crunch, generating a prolonged recession.

It is worth mentioning that the financial channel as well as the balance sheet substitution channel amplify the initial exogenous buildup of FX reserves shock. On the contrary, both channels work as a stabilization mechanism when FX interventions are implemented as a response to external shocks. Figure 10 summarizes the main transmission mechanisms through which FX interventions stabilize financial and macroeconomic volatility.

FIGURE 10. *Stabilization channels of FX interventions*



4.3 *Welfare Analysis*

Welfare Gains/Losses. We look for the optimal and implementable combination of policy rules for the short-term interest rate, i_t , and central bank sterilization bonds, B_t, defined by equations (16) and (45). We follow Schmitt-Grohé and Uribe (2007) and define an implementable policy if it ensures local uniqueness of the rational expectations equilibrium. Furthermore, we define an optimal policy if the contingent plans of consumption, and hours of work associated with a particular implementable monetary policy mix within the class of policy rules considered yield the highest level of lifetime utility. With the latter being conditional to the current state of the economy.

We consider two monetary policy regimes: i) the reference policy regime, denoted by $\mathcal{R}$; and ii) the alternative policy regime, denoted by $\mathcal{A}$. Each policy regime is characterized by its own time-invariant, stochastic-equilibrium allocation. This consists of two policy instruments: the short-term nominal interest rate and the central bank bonds used for sterilizing FX interventions. The benchmark regime, $\mathcal{R}$, is such that each policy rule is calibrated as in the case of the baseline parameterization of the model: $\upsilon_e = 9.7$ for the FX intervention rule, and $\rho_i = 0.7$, $\omega_\pi = 1.5$, and

$\omega_y = 0.125$ for the interest rate rule. Recall that in our baseline model, υ_e, is part of the parameter space estimated by impulse-response matching, while the parametrization of the interest rate rule follows the standard calibration found in the literature. Therefore, neither parameterization in the reference regime is necessarily optimal. In the alternative regime $\mathcal{A}$, we examine a wide range of the policy coefficients for (υ_e, ω_π), including the absence of FX interventions under a flexible exchange rate regime, i.e., $\upsilon_e = 0$.

Note that the set of policy regimes we evaluate changes the dynamics of the model economy, but not its non-stochastic-steady state (welfare measures are conditioned upon the initial non-stochastic steady state of the economy). This ensures that the economy begins from the same initial point under all possible policies.

As a result, the conditional welfare associated with the reference policy regime $\mathcal{R}$ is defined as:

$$\mathbb{W}(\{C_t^{\mathcal{R}}, H_t^{\mathcal{R}}\}_{t\geq 0}) \equiv (1-\beta)\mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma}(C_t^{\mathcal{R}} - \mathcal{H}C_{t-1}^{\mathcal{R}} - \frac{\zeta_0}{1+\zeta}(H_t^{\mathcal{R}})^{1+\zeta})^{1-\gamma}\right]$$

where $\{C_t^{\mathcal{R}}, H_t^{\mathcal{R}}\}_{t\geq 0}$ is the corresponding contingent plan for consumption and hours of work under the policy regime $\mathcal{R}$. Similarly, the conditional welfare associated with the alternative policy regime $\mathcal{A}$ is defined as:

$$\mathbb{W}(\{C_t^{\mathcal{A}}, H_t^{\mathcal{A}}\}_{t\geq 0}) \equiv (1-\beta)\mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma}(C_t^{\mathcal{A}} - \mathcal{H}C_{t-1}^{\mathcal{A}} - \frac{\zeta_0}{1+\zeta}(H_t^{\mathcal{A}})^{1+\zeta})^{1-\gamma}\right]$$

where $\{C_t^{\mathcal{A}}, H_t^{\mathcal{A}}\}_{t\geq 0}$ is the corresponding contingent plan for consumption and hours of work under the policy regime $\mathcal{A}$.

Let ς denote the welfare cost of adopting the policy regime $\mathcal{A}$ instead of the reference policy regime $\mathcal{R}$, under the condition that the economy starts at its non-stochastic steady state at time zero. The parameter ς measures the fraction of the consumption process associated with the reference regime that a household would be willing to give up to be as well off under the alternative policy regime $\mathcal{A}$, as under regime $\mathcal{R}$. Thus, ς is implicitly defined by

$$\mathbb{W}(\{C_t^{\mathcal{A}}, H_t^{\mathcal{A}}\}_{t\geq 0}) = \mathbb{W}(\{(1+\varsigma)C_t^{\mathcal{R}}, H_t^{\mathcal{R}}\}_{t\geq 0}) \tag{53}$$

Hence, if $\varsigma > 0$ there is a welfare gain while if $\varsigma < 0$ then there is a welfare loss under the alternative regime $\mathcal{A}$. We approximate ς up to a second order of accuracy.

Figure 11 displays the welfare gains for different values of the FX intervention policy coefficient relative to the baseline policy regime. Our results indicate that given the Taylor coefficients at their baseline levels, not responding to the real exchange rate by implementing FX interventions, ($\upsilon_e = 0$), would cause a welfare loss of 4.0 percent in terms of consumption. Additionally, Figure 11 shows that the optimal FX intervention response, υ_e, is much higher than its baseline estimated level of $\upsilon_e = 9.71$. If we keep the Taylor coefficients at their baseline levels the optimal FX intervention policy response is $\upsilon_e = 28.1$ which is associated with a welfare gain of 1.2 percent in terms of consumption. Finally, aggressive responses of FX interventions to exchange rate deviations from its steady-state level, υ_e higher than its optimal level, reduce welfare gains (see the region $\upsilon_e > 28.1$ in Figure 6). The latter result is due to higher central bank operational losses.

FIGURE 11. *Welfare gains of FX intervention policy*

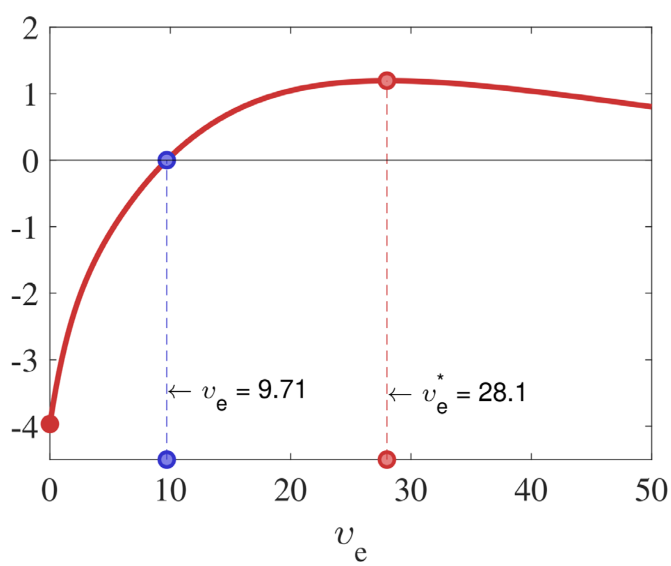


Table 6 displays welfare gains for various combinations of the policy parameters (υ_e, ω_π), relative to the baseline parametrization. Clearly, conditional to external shocks, FX intervention policy together with a standard short-term nominal interest rate policy, improve welfare in a considerable region of the selected policy coefficient space. However, if the Taylor rule coefficient on inflation is much higher than its baseline level, for example ω_π between 3 and 5, the results displayed in Table 6 indicate that a flexible exchange rate regime ($\upsilon_e = 0$) might be welfare-improving relative to the baseline FX intervention coefficient ($\upsilon_e = 9.71$). But we feel that policy coefficients larger than 3 would be difficult to communicate to policymakers. Nevertheless, the optimal FX intervention rule coefficient on deviations of the real exchange rate is non-zero for a wide range of the Taylor rule coefficient on inflation. For example, when $\omega_\pi = 3$, the optimal value for υ_e is around 10. But when $\omega_\pi = 1.25$, the optimal value for υ_e is close to 50.

TABLE 6. *Welfare gains of FX intervention policy*

$\omega_\pi \setminus \upsilon_e$	0.0	2.5	5.0	**Baseline: 9.71**	20.0	30.0	50.0	100.0
1.25	-14.4	-11.0	-8.9	-5.7	-1.9	-0.5	0.1	-0.3
Baseline: 1.50	-4.0	-2.0	-1.1	**0.0**	1.0	1.2	0.8	-0.2
2.00	-0.6	0.7	1.2	1.6	1.8	1.6	0.9	-0.2
3.00	0.5	1.5	1.8	2.1	2.0	1.7	1.0	-0.2
5.00	0.8	1.7	2.0	2.2	2.1	1.7	1.0	-0.3

Note. The parameter ω_π measures the interest rate response to fluctuations in inflation, while υ_e measures the response of FX interventions to real exchange rate deviations. For each combination of (ω_π, υ_e) we compute ς which is defined above and report $\varsigma \times 100\%$. Only external shocks are considered in this simulation.

We consider these results suggest that the active use of FX interventions as an additional monetary policy tool are welfare-improving relative to a flexible exchange rate regime, particularly when the Taylor rule coefficient on inflation is calibrated around standard values. When the latter coefficient is equal to or greater than 3, the flexible exchange rate regime is ranked last in terms of welfare gains, at least for values of $\upsilon_e < 30$. For each value of the Taylor rule coefficient on inflation, the welfare gains of FX interventions under the optimal value for υ_e are substantial, between 1 and 2.2 percent of consumption. This is a large number in business-cycle studies, and it is driven by the financial frictions considered in our baseline framework that generate endogenous UIP deviations for banks and households.

Optimal FX Intervention response under adjustment costs for sterilization operations. What is the optimal response of FX intervention to external shocks? In general, this is a hard question to answer since it involves many aspects of official FX reserve management that are not contemplated in our framework. In this section we give a preliminary answer to this question in a context where the central bank faces adjustment costs when implementing the corresponding sterilization operations associated with FX interventions. We consider that the adjustment cost of FX intervention policy is proportional to the central bank's quasi-fiscal cost. In this case, we assume the central bank's quasi-fiscal deficit considering adjustment costs is given by:

$$CB_t = e^{\tau^{fx}(B_t - B)}(R_t^b - \frac{e_t}{e_{t-1}} R_t^*)B_{t-1}$$

Note that if $\tau^{fx} > 0$, the central bank faces adjustment costs in excess of $(R_t^b - \frac{e_t}{e_{t-1}} R_t^*)B_{t-1}$.The adjustment costs arise because the central bank is sterilizing an FX operation during the period by using the FX policy rule given in Equations (14) and (16), while the quasi-fiscal deficit is a function of the stock of central bank bonds at the beginning of the period. Therefore, when the central bank changes B_t, it must pay an additional cost that is proportional to the quasi-fiscal cost associated with B_{t-1}.

Define S_t as:

$$S_t := \frac{CB_t}{B_{t-1}} = exp(\tau^{fx}(B_t - B))\left(R_t^b - \frac{e_t}{e_{t-1}} R_t^*\right) \Rightarrow \frac{1}{S_t}\frac{\partial S_t}{\partial B_t} = \tau^{fx}$$

consequently, τ^{fx} measures a partial elasticity of S_t with respect to B_t.

As explained above, these adjustment costs are part the central bank's quasi-fiscal deficit implying that a fraction of non-commodity aggregate output is allocated to fund the adjustment costs that the central bank face when sterilizing FX interventions. For clarity, we re-write equation (46) in terms of the adjustment costs the central face when sterilizing FX interventions:

$$Y_t^{nc} - \widehat{REST}_t = C_t + G_t + I_t^{nc} + I_t^c + Y_t^{x,nc}$$

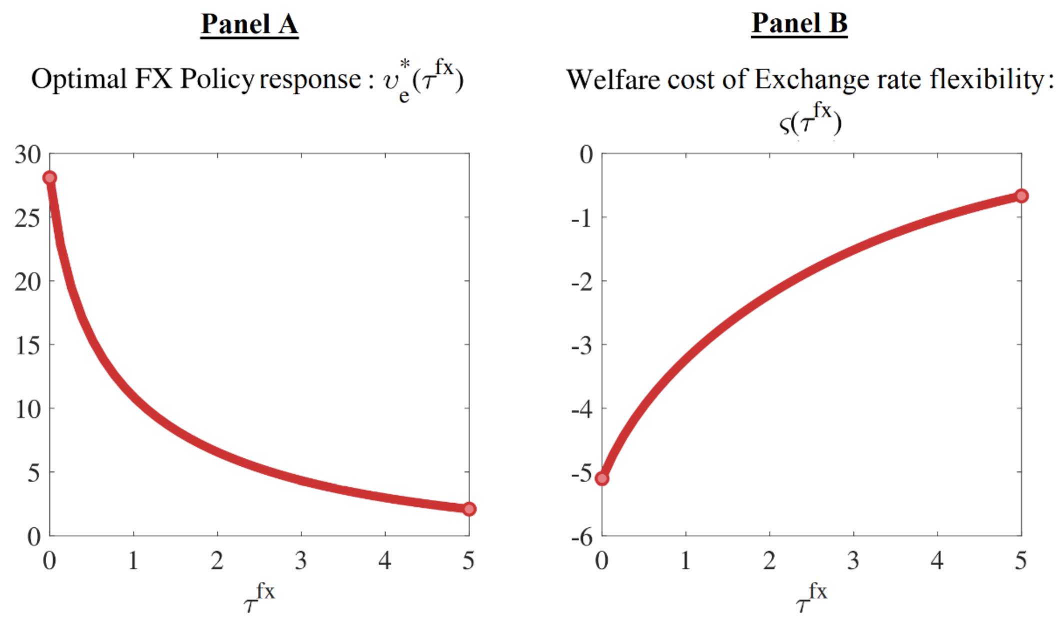
where $\widehat{REST}_t$ *is*

$$\widehat{REST}_t = REST_t + \left(e^{\tau^{fx}(B_t - B)} - 1\right)\left(R_t^b - \frac{e_t}{e_{t-1}} R_t^*\right)B_{t-1}$$

The parameter τ^{fx}measures how costly are deviations of central bank bonds with respect to their steady-state level in terms of aggregate non-commodity output. In our model, changes in the supply of central bank bonds are exclusively associated with the sterilization operations of FX intervention policy. We consider values of τ^{fx} between 0 and 500 basis points. For each value of τ^{fx}, we calculate the optimal size of the real exchange rate coefficient in the FX intervention rule by computing the welfare cost of implementing a flexible exchange rate regime relative to the optimized FX intervention rule. In this case, the reference regime varies with each value of τ^{fx} and it is associated with the optimal value of υ_e while the alternative regime represents the case of exchange rate flexibility, $\upsilon_e = 0$. We assume that the Taylor rule coefficients are calibrated as in the baseline parametrization of the model.

Panel A of Figure 12 displays the optimal FX intervention policy coefficient for a wide range of values for τ^{fx} while Panel B of Figure 12 shows the welfare cost of implementing a flexible exchange rate regime for each value of τ^{fx}. When $\tau^{fx} = 0$, the optimal FX policy response to real exchange rate deviations is $\upsilon_e = 28.1$ as we have already showed in Figure 11. The welfare cost of exchange rate flexibility is around 5 percent of consumption which is 1 percent higher than the corresponding welfare cost obtained for the baseline parametrization of the FX intervention coefficient. More importantly, Figure 12 shows that for higher values of τ^{fx}, the optimized FX intervention policy coefficient decreases monotonically, from $\upsilon_e = 28.1$ to $\upsilon_e = 3.5$. Similarly, we find that the welfare cost of implementing a flexible exchange rate regime decreases when the central bank faces higher adjustment costs when implementing the corresponding sterilization operations associated with FX interventions, that is, higher values of τ^{fx}. Particularly, Panel B of Figure 12, shows that the welfare cost of exchange rate flexibility varies from 5 percent to 1.5 percent of consumption relative to each optimized FX intervention rule. Our results suggest that exchange rate flexibility generate substantial welfare losses when compared to the optimized FX intervention policy regime.

FIGURE 12. *Optimal FX intervention rule*



5. Numerical Experiments under More General Assumptions

5.1 Relaxing Assumptions about the Financial System

We examine the effectiveness of FX interventions as a response to external shocks under more general assumptions. We compare results under the baseline model with the following extensions:[21]

> ***Case 1**: **An economy without financial dollarization**. Intermediate good producers borrow from banks only in domestic currency while households are not allowed to hold deposits with banks that are denominated in foreign currency. This case attempts to extend our results for a typical emerging market economy without financial dollarization such as Chile, Colombia, Mexico, etc.*
>
> ***Case 2**: **The UIP equation holds with equality for households**. Households demand for bank deposits in foreign currency is infinitely responsive to arbitrage opportunities.*

[21] In Appendix B.2, we present results for an additional extension (Case 0) where the three assets that banks can hold enter with equal weights into the incentive compatibility constraint. Therefore, central bank bonds have a higher impact on the total amount of divertible funds and ultimately on banks' lending capacity. As a result, FX interventions are more effective in this case than in our baseline model.

***Case 3**: **The UIP equation holds with equality for banks**. The size of the currency mismatch affecting bankers' ability to divert funds is assumed to be an aggregate measure of the banking system, therefore it is taken as given at the individual bank level.*

In **case 1**, we consider an emerging market economy without financial dollarization, where intermediate good producers borrow from banks only in domestic currency (i.e., $\delta^f = 0$) and households are not allowed to hold foreign currency-denominated deposits with banks. In this case, the only source of foreign currency-denominated funds for the banking system comes from borrowing abroad. The steady state of the model is recalibrated since some endogenous variables, such as R^l and ϕ^{l*}, are no longer part of the equilibrium equations (see Table 7). By the same token, aggregate deposit dollarization refers exclusively to foreign borrowing by banks, implying that in steady state, deposit dollarization is revised from 69.9 percent in the baseline calibration to 23.9 percent. Therefore, aggregate currency mismatch increases at steady state from 17.7 percent to 23.9 percent, leaving the banking system more exposed to exchange rate fluctuations.

TABLE 7. *Steady state equilibrium*

VARIABLE	BASELINE	**Case 1** $\delta^f = D^{*,h} = 0$	**Case 2** $\downarrow \kappa_{D*}$	**Case 3** AGG. x_t
Financial System Rates				
Capital return	8.0	8.2	8.0	8.0
Domestic currency loan's return	6.0	6.1	6.0	6.0
Foreign currency loan's return	4.0	4.1	4.0	4.0
FX bonds return	4.0	3.9	4.0	4.0
Foreign interest rate	1.0	1.0	1.0	4.0
Deposit interest rate	4.0	4.0	4.0	4.0
Bank leverage in B	1.1	1.3	1.1	1.1
Bank leverage in L	3.0	6.7	3.0	3.0
Bank leverage in eL^*	3.0	0.0	3.0	3.0
Currency mismatch	17.7	20.9	17.7	17.7
Credit dollarization	50.0	0.0	50.0	50.0
Deposit dollarization	69.8	23.9	69.8	69.8
RER	1.0	1.0	1.0	1.0
Sectoral Rates				
Commodity/total exports	60.0	59.9	60.0	60.0
Commodity/total investment	16.7	16.8	16.7	16.7
Stock Rates				
Noncommodity capital/GDP	1.9	1.8	1.9	1.9
Commodity capital/GDP	1.3	1.3	1.3	1.3
Stock of capital/GDP	1.8	1.8	1.8	1.8
Foreign reserves/GDP	22.0	21.9	22.6	22.0
Aggregate Demand Rates				
Investment/GDP	18.0	17.7	18.5	18.0
Public consumption/GDP	14.0	13.9	14.4	14.0
Consumption/GDP	60.0	60.3	58.9	60.0
Current account/GDP	-0.0	0.1	-0.0	-0.2
Trade balance/GDP	8.0	8.1	8.2	8.0

Table 9 shows that aggregate volatility under exchange rate flexibility for an economy without domestic financial dollarization (i.e., **case 1**) is significantly lower than the economy with financial dollarization (i.e., baseline model). Bear in mind that both economies face the same external shocks, the only difference being the existence of domestic financial dollarization on both sides of banks' balance sheets. For instance, the volatility of financial variables such as currency mismatch and total credit is reduced by around 80 and 45 percent, respectively. Therefore, the volatility of macroeconomic aggregates such as investment and GDP drop significantly as well. Consequently, our results indicate that—maintaining all else equal—domestic financial dollarization under exchange rate flexibility serves as an amplifier of external shocks, inducing higher aggregate volatility into business cycle fluctuations in EMEs.

Even though, aggregate volatility is substantially reduced under exchange rate flexibility in the economy represented by **case 1**, FX intervention remains effective in further reducing the real exchange rate and macroeconomic volatility. Table 9 shows that the real exchange rate volatility in **case 1** is reduced by around 52 percent when FX interventions are active. The latter decline in real exchange rate volatility is lower than the 62 percent drop obtained in our baseline case. Likewise, the reduction in volatility under FX interventions is also carried through total credit, investment and GDP. Notably, the volatility of the aggregate currency mismatch increases under the FX intervention regime, which is a direct result of not having domestic credit and deposit dollarization. Recall that the only source of foreign currency funding for banks is borrowing from abroad (i.e., external credit lines) which is more sensitive to FX operations (see Figure 5 and Figure 7), implying that currency mismatch absorbs all the volatility associated with it.

TABLE 8. *Aggregate volatility conditional on external shocks*

	Baseline		**Case 1**		**Case 2**		**Case 3**	
	FXI	FER	FXI	FER	FXI	FER	FXI	FER
RER	2.07	5.48	1.72	3.60	1.96	2.06	1.88	1.87
Inflation	0.34	0.74	0.29	0.46	0.26	0.29	0.24	0.22
UIP Dev.	0.30	1.36	0.19	0.16	0.01	0.01	0.00	0.00
GDP	0.86	1.91	0.63	1.42	0.71	0.85	0.65	0.79
Investment	5.03	10.31	3.80	8.95	4.15	4.79	4.09	5.32
Total Credit	1.04	3.88	1.35	1.89	0.52	0.68	1.45	2.28
Currency Mismatch	1.79	4.35	2.14	0.86	0.33	0.38	2.20	2.13
Consumption	0.71	0.94	0.52	0.72	0.35	0.40	0.19	0.20
Labor	0.40	0.51	0.40	0.42	0.26	0.28	0.29	0.20

Note. Standard deviations for major aggregate variables. FXI and FER denote Foreign Exchange Intervention and Flexible Exchange Rate policy regime respectively. The computation consider the external block as the only source of aggregate volatility and it is based on 2500 replications of 120 periods simulated trajectories.

Next, we study the role of the real exchange rate smoothing channel by changing the assumptions that affect the presence of endogenous UIP deviations. For FX operations to be effective in changing the real exchange rate dynamics, both households and banks must face limits to arbitrage between domestic and foreign currency-denominated assets and liabilities. If there were no limits to arbitrage for at least one of these agents, then the effectiveness of FX intervention policy might change drastically. To see this, recall that there are two endogenous UIP deviations in the model given by

$$\mathbb{E}_t \Lambda_{t+1}(R_{t+1} - \frac{e_{t+1}}{e_t} R^*_{t+1}) = \kappa_{D*}(\overline{D}^{*,h} - D^{*,h}_t) \qquad \text{(UIP Households)}$$

$$\mathbb{E}_t \Omega_{t+1}(R_{t+1} - \frac{e_{t+1}}{e_t} R^*_{t+1}) = \frac{\lambda^b_t}{1+\lambda^b_t}(\frac{l_t + \varpi^* e_t l^*_t + \varpi^b b_t}{l_t + e_t l^*_t + b_t}) \frac{d\Theta(\tilde{x}_t)}{dx^{\text{bank}}} \qquad \text{(UIP Banks)}$$

In particular, when households are allowed to engage in frictionless arbitrage between domestic and foreign currency-denominated bank deposits, there are no endogenous excess returns in equilibrium, i.e., the right-hand side of eq. (UIP Households) converges to zero as κ_{D*} goes to zero. We examine an approximation of this economy in **case 2** (see Table 7) where κ_{D*} is fixed at a sufficiently small but positive number. Consequently, the UIP condition for households holds with a constant premium with no endogenous deviations from it.

Our results indicate (see Table 9) that in **case 2**, FX interventions do not reduce real exchange rate volatility relative to the flexible exchange rate regime. Therefore, the real exchange rate smoothing channel is muted. Table 9 also shows that there is a smaller effect of FX interventions on aggregate volatility that emerges from the *balance sheet substitution channel* due to the sterilization leg of FX interventions. In particular, the volatility of total credit is reduced by around 23 percent, while GDP and investment volatility is reduced by 17 and 13 percent, respectively. Although the incentive compatibility constraint for banks still binds (i.e., $\lambda^b > 0$), implying endogenous deviations from the bank's UIP condition, it turns out to be not quantitatively significant as long as households are able to arbitrage without cost.

Likewise, in **case 3**, banks do not internalize the effects of borrowing in foreign currency on the aggregate currency mismatch of the banking system. In terms of the UIP equation for banks, **case 3** implies the last term is zero:

$$\frac{d\Theta(\tilde{x})}{dx^{bank}} = \frac{d\Theta(x_t^{industry})}{dx^{bank}} = 0$$

Therefore, in **case 3** the banker's ability to divert funds depends on an industry measure of currency mismatch instead of a measure at the bank level. Then, banks are indifferent between borrowing from domestic depositors and from abroad, implying that the standard UIP condition holds without any endogenous risk premium. Notably in this case, even though the incentive constraint binds (i.e., $\lambda^b > 0$), the response of the real exchange rate to external shocks is similar under the FX intervention policy and the flexible exchange rate regimes. Table 9 shows that FX interventions do not reduce the volatility of the real exchange rate. Therefore, as in **case 2**, the reduction of aggregate volatility of credit, GDP, and investment (by about 36, 18, and 23 percent, respectively) emerges mainly from the *balance sheet substitution channel*. This result, together with **case 2**, differ from Céspedes et al. (2017) and Chang (2019) where FX interventions are irrelevant only when the incentive compatibility constraint does not bind (i.e., $\lambda^b = 0$ in (UIP Banks)). Our result is due to the indeterminacy of banks' liability composition that occurs when banks do not internalize the effect of currency mismatch on financial constraints.[22]

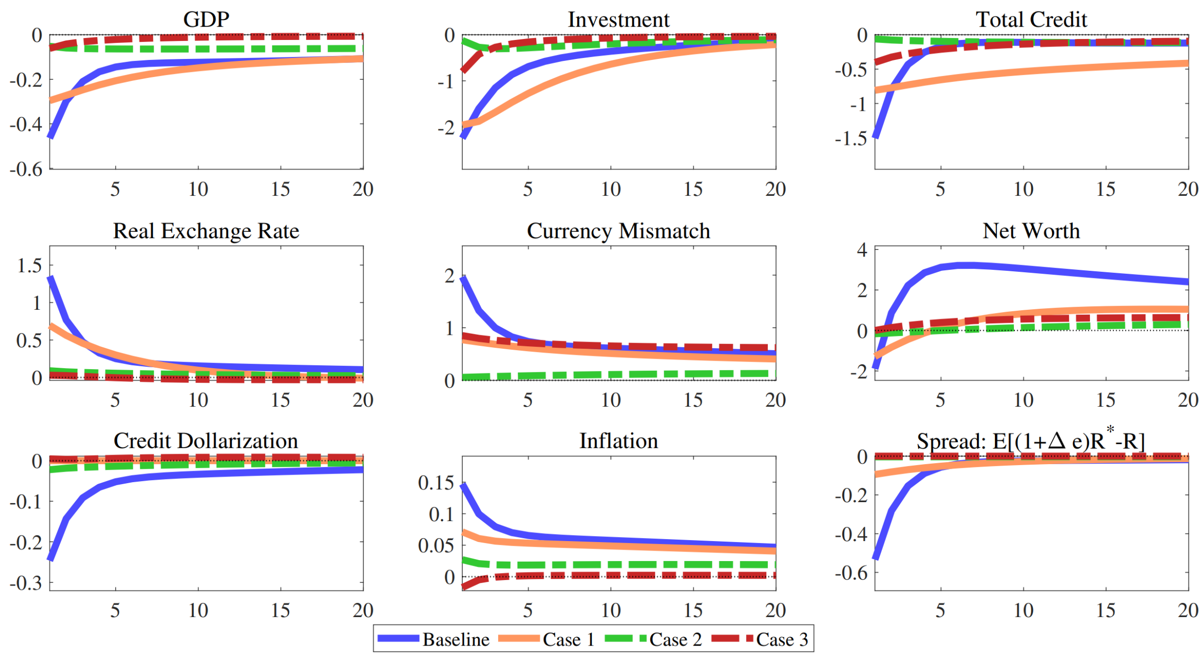
Figure 13 displays responses to a sufficiently persistent unanticipated purchase of official FX reserves for each of the extensions considered in this section, including our baseline model. Under **cases 2** and **3** practically show no real exchange rate response. As stated above, the latter result is explained because, in **cases 2** and **3**, the UIP condition for banks or households holds with equality even though the incentive constraint for banks still binds (i.e., there are no endogenous deviations from UIP). Therefore, FX interventions are neutral with respect to real exchange rate dynamics. Figure 13 also shows that the implications of muting the real exchange rate smoothing channel are not trivial for the rest of the aggregate variables such as credit, GDP, investment, and inflation.

Table 12 (in Appendix B) shows that if FX interventions are switched off, $v_e = 0$, and the Taylor rule parameters are kept as in the baseline calibration of our model, when the economy does not face domestic financial dollarization, welfare diminishes by less than in our baseline economy, i.e., the welfare loss associated with a flexible exchange rate regime is 2.4 percent in

[22] Technically, there is one more case that we could examine in our model: when the agency problem does not depend on any currency mismatch measure, i.e., $\Theta(x)$ equals some positive constant for any x. However, we argue that this case implies qualitative consequences similar to those of **case 3**. Results are available upon request.

terms of consumption (see Table 13 in Appendix B). Finally, when FX interventions are neutral in smoothing the real exchange rate response to external shocks (as in **cases 2** and **3**), welfare gains are remarkably close to zero under exchange rate flexibility when compared to the welfare loss in our baseline economy (0.1-0.2 percent instead of 6.2 percent; see Table 14 and Table 15 in Appendix B).

FIGURE 13. *Responses to a Persistent Purchase of FX Reserves under different generalizations*



Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state.

5.2 Smoothing Exchange Rate Dynamics by Using the Taylor Rule

In this subsection, we compare the performance of two policy regimes which both implement a managed float for a small open economy: i) Our baseline policy regime with two monetary instruments, i_t and B_t, which are determined by the following policy rules:

$$lnB_t = lnB - v_e(lne_t - ln_e)$$

and

$$i_t - i = \rho_i(i_{t-1} - i) + (1 - \rho_i)\left[\omega_\pi \pi_t + \omega_y ln\left(\frac{\text{GDP}_t}{\text{GDP}}\right)\right];$$

and ii) An alternative policy regime with the policy interest rate, i_t, as the only monetary instrument following an extended Taylor rule that responds not only to inflation and the output gap, but also to deviations of the real exchange rate with respect to its steady state value:[23]

$$i_t - i = \rho_i(i_{t-1} - i) + (1 - \rho_i)[\omega_\pi \pi_t + \omega_y ln\left(\frac{\text{GDP}_t}{\text{GDP}}\right) + \omega_e(lne_t - lne)], \tag{54}$$

We compare both policy regimes in terms of conditional macroeconomic volatility to external shocks and in terms of welfare. Table 9 shows simulated standard deviations for major macroeconomic variables under both policy regimes along with exchange rate flexibility. We consider two different calibrations for the extended Taylor rule, $\omega_e = 0.1$ and $\omega_e = 1$, while keeping the rest of its parametrization as in the baseline scenario.

Our results suggest that in our framework both ways of having a managed float are successful in reducing the exchange rate volatility conditional to external shocks. When the central bank implements FX interventions together with a standard Taylor rule as in our baseline model, the exchange rate volatility is reduced from 5.5 percent under exchange rate flexibility to 2.1 percent. On the other hand, when the central bank operates with the extended Taylor rule, the decline obtained in the exchange rate volatility depends on the response of the monetary policy rate to real exchange rate deviations from steady state. Depending on the value of ω_e, the central bank may be able to manage exchange rate fluctuations to the point of reaching a lower standard deviation than in the FXI regime, as can be seen in the third column of Table 9. Notably, the main difference between both managed floating regimes is the volatility of investment relative to exchange rate flexibility. Under the extended Taylor rule, the monetary policy interest rate partially absorbs the volatility of the real exchange rate which ultimately is transmitted to investment volatility. In Table 9, the volatility of investment is considerably higher when the central bank manages the exchange rate with the extended Taylor rule, failing to smooth the response of investment to external shocks relative to the flexible exchange rate regime. Moreover, under the

[23] In line with the spirit of analyzing distinct policy rules, but not necessarily related to the exercise in this section, in Appendix B.4 we present responses to external shocks when the central bank FX intervention rule respond to real depreciation or to the interest rate spread between foreign and domestic interest rates instead of responding to real exchange rate deviations. We use the following FX intervention rule:

$$lnB_t = (1 - \rho_B)lnB + \rho_B lnB_{t-1} - \upsilon_e(lne_t - lne) - \upsilon_{\Delta e}\Delta e_t - \upsilon_{\text{spread}}\left[\left(\frac{e_t}{e_{t-1}}R_t^* - R_t\right) - (R^* - R)\right]$$

We obtain very similar results with both FX intervention rules under a proper calibration of these different types of FX rule. But we suggest that an FX intervention rule in terms of interest rate spreads is not as implementable as an FX intervention rule that responds to exchange rate deviations from steady state.

baseline parametrization of the model the conditional volatility for GDP, consumption, total credit and currency mismatch is lower under the FXI regime than when the central bank uses the extended Taylor rule to manage exchange rate responses (see Table 9).

TABLE 9. *Aggregate volatility conditional on external shocks*

	FER	Extended Taylor (1)	Extended Taylor (2)	FXI
RER	5.49	3.79	1.22	2.09
Inflation	0.74	0.11	1.94	0.34
UIP Dev.	1.35	1.01	0.59	0.30
GDP	1.92	1.79	1.47	0.86
Investment	10.32	13.86	20.83	5.07
Total Credit	3.87	3.46	2.68	1.04
Currency Mismatch	4.35	3.31	1.76	1.79
Consumption	0.93	0.93	0.92	0.71
Labor	0.51	0.58	1.88	0.40

Note. "FER" is the abbreviation for the *Flexible Exchange Rate* regimen, "Extended Taylor (1)" for the regime where the alternative Taylor rule is implemented with $\omega_e = 0.1$ and $\upsilon_e = 0$, "Extended Taylor (2)" refers to the alternative Taylor rule with $\omega_e = 1$ and $\upsilon_e = 0$, and "FXI" is the abbreviation for *Foreign Exchange Intervention* regimen with the standard Taylor rule, $\omega_e = 0$. The computation only considers external shock volatilities and is based on 2500 replications of 120 periods simulated trajectories.

As in Section 4.3, our welfare analysis defines the welfare gain/loss, ς_{cond} by

$$\mathbb{W}(\{C_t^{\mathcal{A}}, H_t^{\mathcal{A}}\}_{t\geq 0}) = \mathbb{W}(\{(1+\varsigma_{cond})C_t^{\mathcal{R}}, H_t^{\mathcal{R}}\}_{t\geq 0}) \tag{55}$$

where the $\mathcal{R}$ regime refers to the baseline policy regime with, $\omega_\pi = 1.50$, $\omega_e = 0$, and $\upsilon_e = 9.71$, while in this case, the alternative regime $\mathcal{A}$ is specified by a combination of different values for ω_e and ω_π for the extended Taylor rule regime.

Table 10 shows welfare losses for the extended Taylor rule regime. Our results indicate that, for a non-trivial region of the parameter space, even when the central bank has the policy interest rate as the only monetary instrument responding to real exchange rate deviations $\omega_e \geq 0$, there are significant welfare losses relative to our baseline model ($\omega_\pi = 1.5$, $\omega_e = 0.0$, $\upsilon_e = 9.7$). Particularly, when compared to the regime identified by ($\omega_\pi = 1.5$, $\omega_e = 1.5$, $\upsilon_e = 0$), Table 10 shows that for a subspace of the policy parameter region of the extended Taylor rule, increasing the response of the policy rate to real exchange rate deviations from steady state reduces the welfare loss (i.e., it is welfare improving) but not as much as in the FXI policy regime. For example, when $\omega_\pi = 1.5$ as in our baseline calibration, increasing ω_e from 0.0 to 0.5 reduces welfare losses by 2.7 percentage points in consumption. Under the baseline policy regime, increasing the response of FX interventions to exchange rate deviations from steady-state from 0.0 to 5.0 reduces welfare losses by 2.9 percent points (see Table 6 in Section 4.3). Moreover, under the extended Taylor rule regime, increasing ω_e may produce indeterminacy for the rational expectation equilibrium.

TABLE 10. *Welfare analysis: $\varsigma \times 100\%$, Extended Taylor Rule regime*

$\omega_\pi \setminus \omega_e$	**Baseline: 0.00**	0.25	0.5	0.75	1.0
1.25	-14.4	0.8	-2.5	-5.8	-8.4
Baseline: 1.50	**-4.0**	1.1	-1.3	-4.1	-6.4
2.00	-0.6	1.2	0.0	-1.8	-3.7
3.00	0.5	1.2	0.9	0.1	-1.0
5.00	0.8	1.2	1.2	1.0	0.6

Note. The parameter ω_π controls the policy rate response to fluctuations in inflation. Parameter ω_e measures the response of an alternative Taylor rule to real exchange rate deviations:

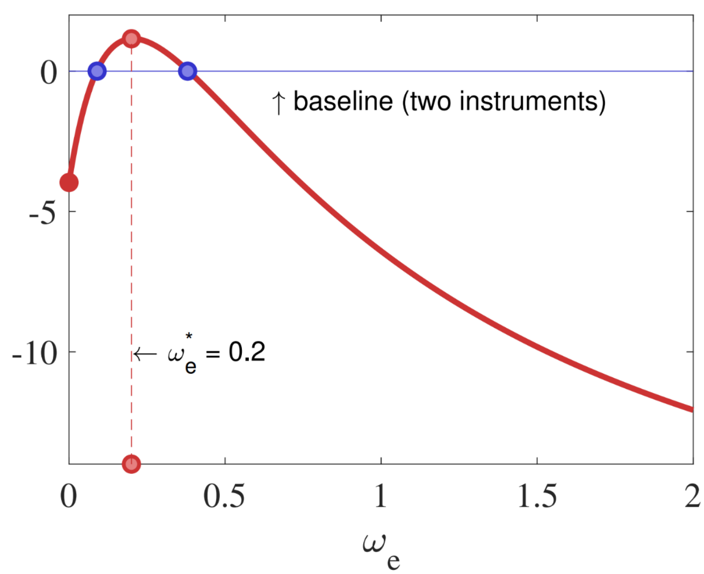
$$i_t - i = \rho_i(i_{t-1} - i) + (1 - \rho_i)\left[\omega_\pi \pi_t + \omega_y ln\left(\frac{\text{GDP}_t}{\text{GDP}}\right) + \omega_e(lne_t - lne)\right]$$

For each combination of (ω_π, ω_e) we compute ς which is defined above eliminating the response of FX interventions to real exchange rate ($\upsilon_e = 0$) in the $\mathcal{A}$ regime and keeping our baseline as the $\mathcal{R}$ regime. Only external shocks are considered.

The results displayed in Table 10 indicates that the policy regime with policy interest rate as the only monetary instrument (extended Taylor regime) can lead to welfare increments relative to the baseline model. To examine this possibility, Figure 14 plots the welfare gains from increasing ω_e but keeping the inflation response on its baseline level $\omega_\pi = 1.5$. In this case, the extended Taylor regime leads to welfare increments whenever ω_e lies between 0.09 and 0.38, otherwise the extended Taylor rule regime leads to significant welfare losses.

Does this result suggest that the extended Taylor regime can be a good substitute to the baseline regime? The quick answer is not necessarily. In fact, keeping $\omega_\pi = 1.5$, the maximum welfare gain under the extended Taylor regime is 1.15 in terms of consumption. The latter is attained at $\omega_e = 0.2$, while the maximum welfare gain under the FXI regime is 1.2 of consumption, which is attained at $\upsilon_e = 28.1$ (see Section 4.3). Hence, although the extended Taylor regime can potentially lead to welfare increments relative to our two-instrument baseline regime, it cannot do better than the optimal FX intervention rule. Moreover, the interval in which the extended Taylor rule is superior to the baseline regime is small enough to suspect about implementation difficulties of this regime at this interval. For instance, if the central bank's mistakes occur with non-zero probability, then it is more likely to obtain welfare losses under the extended Taylor regime than under the optimal FX intervention rule regime. However, we believe that this is an important question and deserves more discussion in future research.

FIGURE 14. *Welfare gain of extended Taylor rule*



6. Concluding Remarks

In this paper we have proposed a macroeconomic model with financial frictions for a small open economy to analyze and quantify the effectiveness of FX interventions in stabilizing the impact of external shocks. FX interventions are modeled as an unconventional monetary policy tool that operates simultaneously with the conventional policy rate tool. More specifically, in our model FX interventions are considered a balance sheet policy induced by an agency problem between banks and their investors (i.e., domestic depositors and foreign lenders). Three key assumptions are important for our results. First, the severity of banks' agency problem depends directly on a measure of currency mismatch at the bank level. Second, the banking system is partially dollarized on both sides of its balance sheet and exposed to potential currency mismatches. On one hand, intermediate good producers must borrow a bundle of loan services from banks in order to produce. The composition of this bundle consists of a combination of domestic and foreign currency-denominated loans. On the other hand, households are allowed to hold deposits with banks that are denominated in domestic and foreign currency. But we introduce limits on household foreign currency-denominated deposits as a way to capture incomplete arbitrage. Third, FX intervention is such that the central bank leans against the wind with respect to exchange rate fluctuations but in a sterilized manner.

Our results shed light on the transmission mechanism of FX interventions. In particular, we highlight two reinforcing effects when responding to external shocks: the exchange rate smoothing channel and the balance sheet substitution channel or crowding-out effect on bank lending. The former channel is active whenever banks and households are not able to seize arbitrage opportunities between domestic and foreign currency-denominated deposits and assets, implying endogenous deviations from UIP. Instead, if either banks or households are able to engage in frictionless arbitrage between domestic and foreign currency-denominated asset returns, the standard UIP equation holds and this channel is no longer active. On the other hand, the balance sheet substitution channel stems from the sterilization operation associated with FX interventions, which modifies the supply of central bank bonds in banks' balance sheet and, with it, their asset composition. Our quantitative results suggest that the latter channel is less significant than the former.

An interesting result arises when banks do not internalize the effects of borrowing in foreign currency on the aggregate currency mismatch of the banking system. In this case, banks are indifferent between borrowing from domestic depositors and from abroad, implying that the standard UIP condition holds without any endogenous risk premium. As a result, FX interventions are less effective in stabilizing the economy in the presence of external shocks. Notably in this case, even though the incentive constraint binds the response of the real exchange rate to external shocks is the same under FX interventions and exchange rate flexibility. This result differs from Céspedes et al. (2017) and Chang (2019), where FX interventions are irrelevant only when the incentive compatibility constraint does not bind. In sum, in our framework, for FX interventions to affect significantly the real exchange rate and excess returns along with the aggregate equilibrium of the economy, limits to arbitrage between domestic and foreign currency-denominated assets and liabilities must be present for both households and banks.

We consider that the financial friction view of FX interventions needs further research. For instance, it differs from the unconventional monetary policy framework for closed economies in several ways. First, FX interventions have been implemented effectively even in normal times in EMEs, contrary to the unconventional monetary policy tools studied in the context of closed economies. In the latter case, once the effective lower bound is reached, unconventional tools may be deployed. Second, what really matters for EMEs is how tight financial constraints are, and not necessarily if they bind or not. Third, in practice, the communication of FX interventions is at odds with the communication of unconventional policies in closed economies. For example, it seems that there is much less forward guidance associated with FX interventions than with QE or LSAP. Finally, the effective lower bound for EMEs may not only be related to the nominal interest rate, but also to a non-negative amount of official FX reserves needed to implement FX interventions within an inflation-targeting regime.

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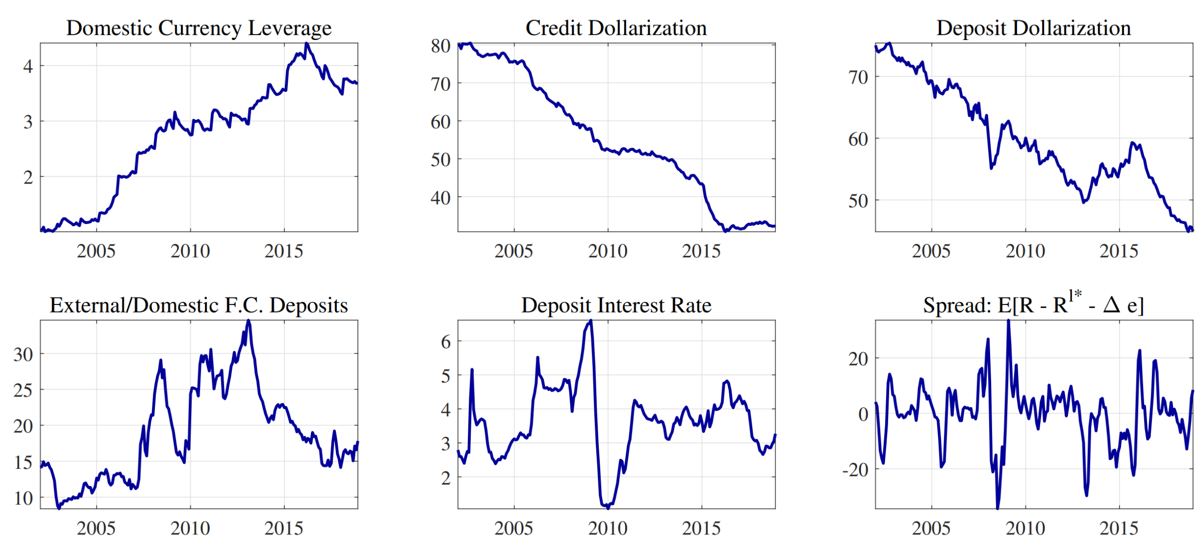
Appendix A. Parametrization

We set the steady-state targets based on Peruvian banking system data. First, calibrate the consolidated balance sheet of the banking system in the model using data for Peru to obtain historical averages for the aggregate currency mismatch level and foreign currency liabilities as a fraction of total assets. We use data on domestic currency credit for L_t, dollar-denominated credit for L_t^* and total banking investment for B_t. We use data on banks' net worth for N_t and the sum of foreign currency deposits and external liabilities for measuring D_t^*. Figure 15 plots the evolution of the bank's balance sheet composition that we used to fix the model's steady-state variables.

We calibrate the consolidated balance sheet of the banking system in the model using data for Peru to obtain historical averages for the aggregate currency mismatch level and foreign currency liabilities as a fraction of total assets. We use data on domestic currency credit for L_t, foreign currency-denominated liabilities for $L_t^*$and total banking investments for B_t. Additionally, we use data on banks' net worth for $N_t$and the sum of foreign currency deposits and external liabilities for measuring D_t^*.

FIGURE 15. *Bank's balance sheet composition*



Note. We use data on domestic currency credit for L_t, dollar denominated credit for L_t^* and total banking investment for B_t. We use data on banks' net worth for N_t and the sum of foreign currency deposits and external liabilities for measuring D_t^*.

Moreover, we use the average of domestic (foreign) currency prime, corporate, and big company loans' interest rate as our measure of domestic (foreign) currency lending return.

Similarly, Figure 16 presents the aggregate real ratios used to fix the demand side steady state of the economy.

Finally, Table 11 summarizes the baseline parametrization used to fix some steady state targets.

FIGURE 16. *Real aggregate ratios*

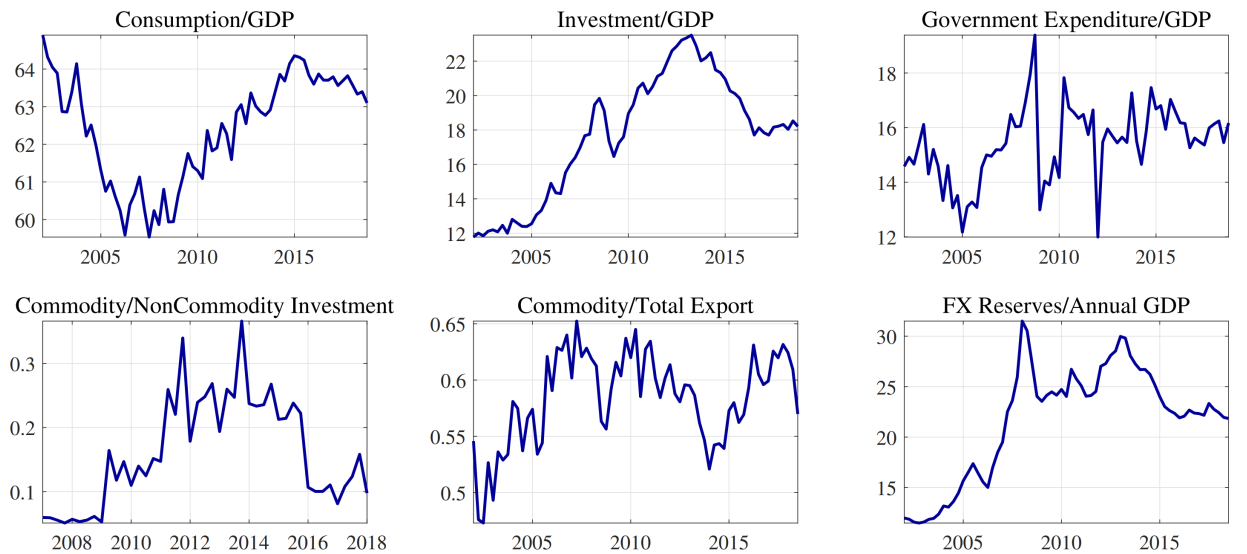


TABLE 11. *Targeted parametrization*

Targets			Parameters		
Description	**Variable**	**Value**	**Description**	**Parameter**	**Value**
Banking Sector					
Foreign Interest Rate	R^*	$1.01^{1/4}$	Foreign Rate	R^*	1.01
Domestic Interest Rate	R	$1.04^{1/4}$	Subjective Discount Factor	β	0.99
Domestic C. Loans Return	R^l	$1.06^{1/4}$	Foreign C. Loans Participation	ϖ^*	1.42
Foreign C. Loans Return	R^{l*}	$1.04^{1/4}$	Bonds Loans Participation	ϖ^b	0.21
CB Bond Return	R^b	$1.04^{1/4}$	Banker's Start-up Transfers	ξ	3.36e-04
Foreign Liability to Asset	$\frac{eD^*}{A}$	53.5%	Moral Hazard 1	θ	0.57
Domestic C. Leverage	$\frac{L}{N}$	3.50	Moral Hazard 2	$\varkappa$	7.17
Credit Dollarization	$\frac{eL^*}{L+eL^*}$	42.5%	DC Bias in Loans	δ^f	0.50
Non-Banking Sector					
Capital Return	R^k	$1.08^{1/4}$	Loans Aggregator Scale	A^e	1.99
Worked Hours	H	1/3	Labor Disutility Scale	ζ_0	3.12
Real Exchange Rate	e	1	Foreign Output	Y^*	0.15
Commodity Price	p^{wc}	1	Commodity Price	p^{wc}	1.00
GDP	GDP	1	Non Commodity Productivity	A^{nc}	0.50
Commodity to Total Export	$\frac{Y^{x,c}}{Y^x}$	60%	Commodity Returns to Scale	α^c	0.18
Commodity to Non Commodity Inv.	$\frac{I^c}{I^{nc}}$	20%	Commodity Sector Productivity	A^c	0.22
Share of Capital financed by households	$\frac{S}{K^{nc}}$	20%	Capital Share	α^k	0.29
Share of Foreign FC Deposits	$\frac{D^{f,*}}{D^+}$	20%	Ef. Household's Capital	$\overline{S}$	0.63
Consumption to GDP	$\frac{C}{GDP}$	60%	Ef. Household's FC Deposits	$\overline{D}^{h,*}$	2.75
Investment to GDP	$\frac{I}{GDP}$	18%	Imported Input Share	α^m	0.33
Gov. Purchases to GDP	$\frac{G}{GDP}$	14%	Gov. Expenditure	G	0.14
FX Reserves to GDP - Anual	$\frac{eF}{4\times GDP}$	22%	CB Bonds	B	0.88

Appendix B. Additional Tables and Figures

B.1 Additional Welfare Analysis Tables

TABLE 12. *Welfare analysis: $\varsigma \times 100\%$ - Case 1*

$\omega_\pi \setminus v_e$	0.0	2.5	5.0	**Baseline: 9.71**	20.0	30.0	50.0	100.0
1.25	-8.6	-7.2	-6.1	-4.4	-2.1	-1.1	-0.7	-1.1
Baseline: 1.50	-1.5	-0.9	-0.5	**0.0**	0.5	0.5	0.0	-1.0
2.00	0.6	0.9	1.0	1.2	1.2	0.9	0.2	-1.0
3.00	1.1	1.3	1.5	1.5	1.4	1.0	0.2	-1.0
5.00	1.3	1.5	1.6	1.6	1.4	1.0	0.2	-1.0

Note. The parameter ω_π controls the policy rate response to fluctuations in inflation. Parameter v_e measures the response of FX interventions to real exchange rate deviations. For each combination of (ω_π, v_e) we compute ς_{cond} which is defined above. Only external shocks are considered.

TABLE 13. *Welfare analysis: $\varsigma \times 100\%$ - Case 2*

$\omega_\pi \setminus v_e$	0.0	2.5	5.0	**Baseline: 9.71**	20.0	30.0	50.0	100.0
1.25	-3.2	-3.0	-2.7	-2.4	-2.1	-2.1	-2.8	-4.8
Baseline: 1.50	-0.2	-0.1	-0.0	**0.0**	-0.2	-0.7	-2.0	-4.9
2.00	0.7	0.7	0.7	0.7	0.3	-0.2	-1.7	-4.9
3.00	0.9	0.9	0.9	0.8	0.5	-0.1	-1.6	-4.8
5.00	1.0	1.0	1.0	0.9	0.5	-0.0	-1.6	-4.7

Note. The parameter ω_π controls the policy rate response to fluctuations in inflation. Parameter v_e measures the response of FX interventions to real exchange rate deviations. For each combination of (ω_π, v_e) we compute ς_{cond} which is defined above. Only external shocks are considered.

TABLE 14. *Welfare analysis: $\varsigma \times 100\%$ - Case 3*

$\omega_\pi \setminus v_e$	0.0	2.5	5.0	**Baseline: 9.71**	20.0	30.0	50.0	100.0
1.25	-2.6	-2.6	-2.6	-2.5	-2.5	-2.6	-2.7	-2.3
Baseline: 1.50	-0.0	-0.0	-0.0	**0.0**	-0.0	-0.1	-0.2	0.1
2.00	0.6	0.6	0.6	0.6	0.6	0.6	0.5	0.8
3.00	0.8	0.8	0.8	0.8	0.8	0.8	0.7	1.0
5.00	0.8	0.8	0.9	0.9	0.9	0.8	0.8	1.1

Note. The parameter ω_π controls the policy rate response to fluctuations in inflation. Parameter v_e measures the response of FX interventions to real exchange rate deviations. For each combination of (ω_π, v_e) we compute ς_{cond} which is defined above. Only external shocks are considered.

B.2 Figures and Tables for Case 0: Perfect Substitution among Banks' Assets

The parametrization of the baseline model implies that central bank bonds are harder to deviate relative to loans (i.e.,$\varpi^* > 1 > \varpi^b$). Since central bank bonds are the only sterilization instrument that the central bank is able to use, the role of FX interventions in mitigating the impact of external shocks is limited by the value of ϖ^b. In **case 0**, bank assets enter the incentive constraint with equal weights (i.e., $\varpi^* = \varpi^b = 1$). In other words, domestic currency loans and central bank bounds become perfect substitutes, as in Chang (2019). Hence, central bank bonds have a higher impact on the total amount of divertible funds and ultimately on banks' lending capacity. As a result, FX interventions are more effective as an external shock absorber in this case than in our baseline model.

TABLE 15. *Aggregate volatility with perfect substitution in bank's assets*

	FXI	FER	Δ%
RER	1.57	5.22	-70
Inflation	0.26	0.72	-64
UIP Dev.	0.35	2.40	-86
GDP	0.69	1.70	-59
Investment	3.83	8.16	-53
Total Credit	0.60	3.32	-82
Currency Mismatch	1.15	3.64	-68
Consumption	0.64	0.92	-31
Labor	0.31	0.69	-56

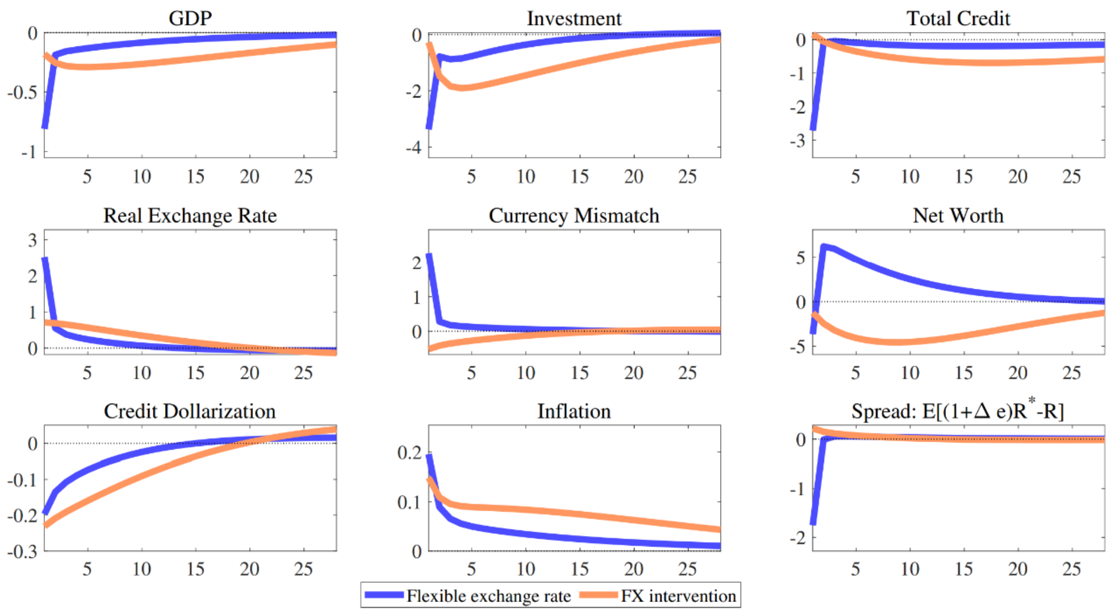
Note. Standard deviations for major aggregate variables. FXI and FER denote Foreign Exchange Intervention and Flexible Exchange Rate policy regime respectively. The computation consider the external block as the only source of aggregate volatility and it is based on 2500 replications of 120 periods simulated trajectories.

TABLE 16. *Welfare analysis: $\varsigma \times 100\%$ - Perfect substitutes in bank's assets*

$\omega_\pi \setminus v_e$	0.0	2.5	5.0	**Baseline: 9.71**	20.0	30.0	50.0	100.0
1.25	-15.6	-11.3	-8.5	-4.6	-1.1	-0.1	0.4	0.4
Baseline: 1.50	-5.4	-2.5	-1.3	**0.0**	1.0	1.1	1.0	0.6
2.00	-2.0	0.1	0.7	1.3	1.6	1.4	1.1	0.6
3.00	-1.0	0.8	1.3	1.6	1.7	1.5	1.1	0.6
5.00	-0.6	1.0	1.4	1.7	1.7	1.5	1.1	0.6

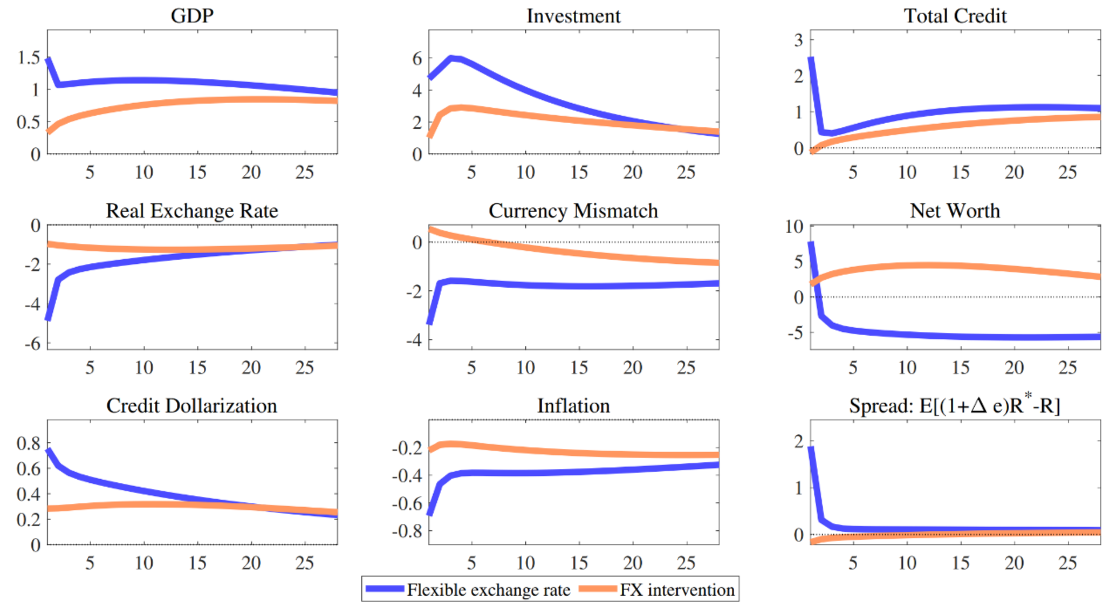
Note. The parameter ω_π controls the policy rate response to fluctuations in inflation. Parameter v_e measures the response of FX interventions to real exchange rate deviations. For each combination of (ω_π, v_e) we compute ς_{cond} which is defined above. Only external shocks are considered.

FIGURE 17. *Foreign interest rate shock: Perfect substitution in bank's assets*



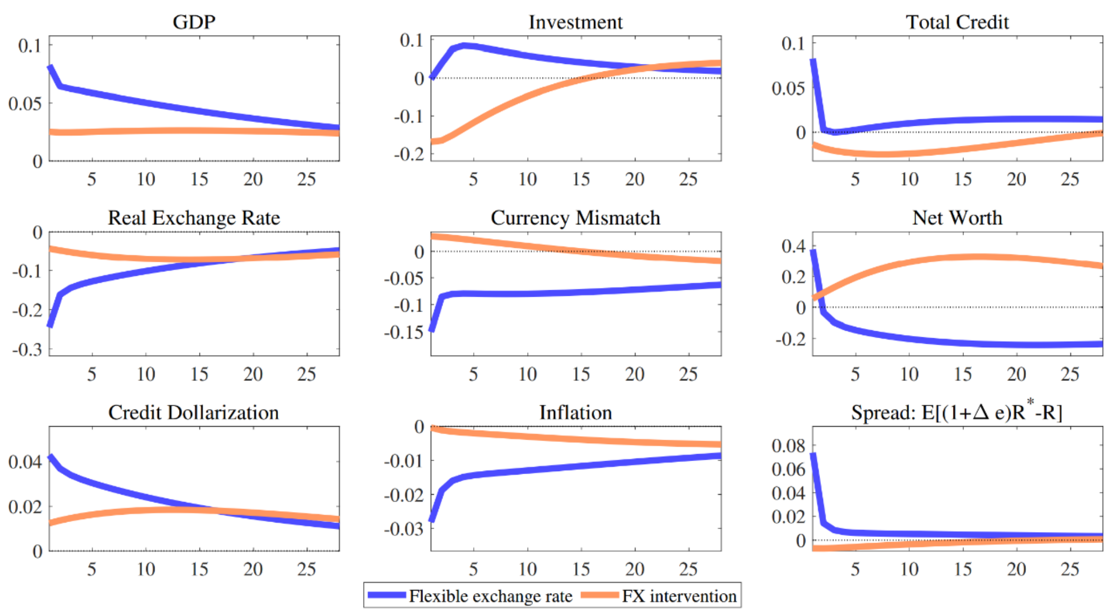
Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^*_{t+1} - R_{t+1}\right]$ measures the relative cost of borrowing in foreign currency from the point of view of banks.

FIGURE 18. *Commodity price shock: Perfect substitution in bank's assets*



Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^*_{t+1} - R_{t+1}\right]$ measures the relative cost of borrowing in foreign currency from the point of view of banks.

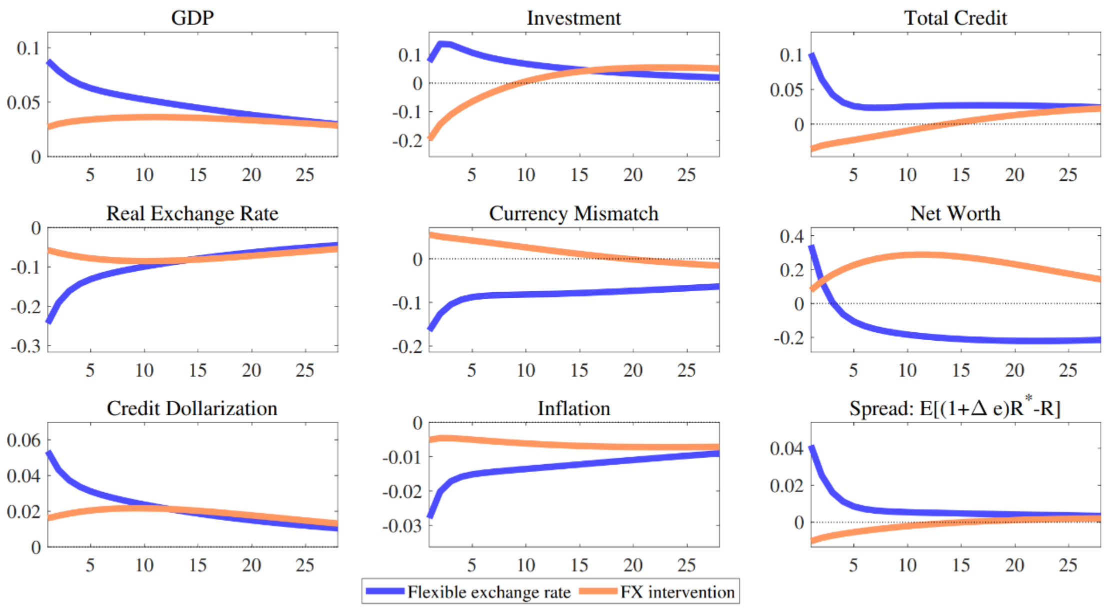
FIGURE 19. *Global demand shock: Perfect substitution in Bank's assets*



Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^*_{t+1} - R_{t+1}\right]$ measures the relative cost of borrowing in foreign currency from the point of view of banks.

B.3 Figures for Foreign Demand Shock

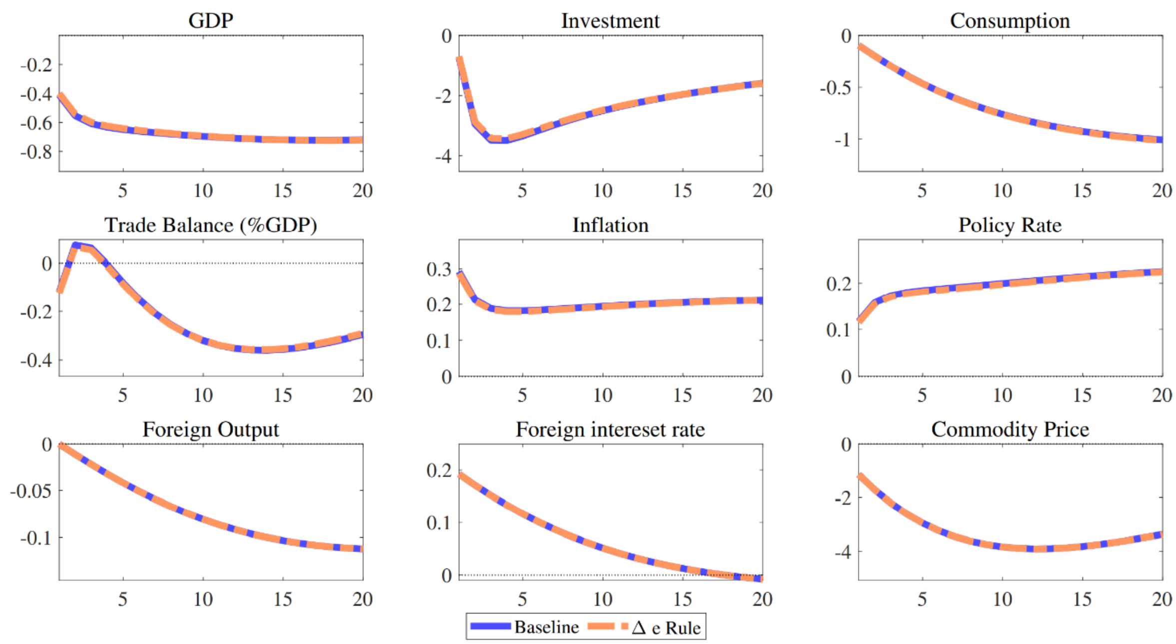
FIGURE 20. *Foreign demand shock in the baseline economy*



Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^*_{t+1} - R_{t+1}\right]$ measures the relative cost of borrowing in foreign currency from the point of view of banks.

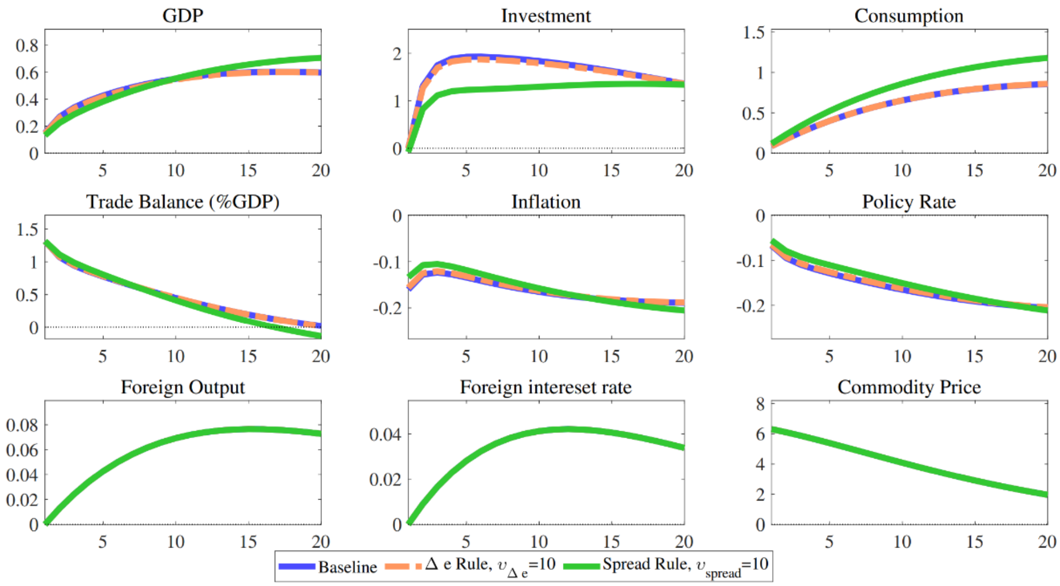
B.4 Distinct FX Intervention Rules

FIGURE 21. *Foreign interest rate shock*



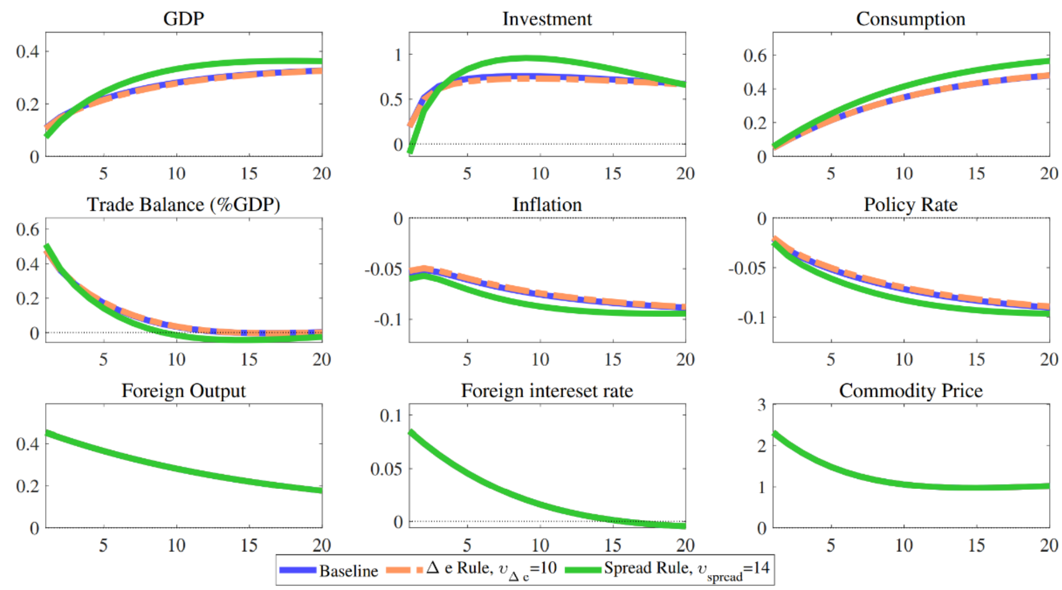
Note. "Δe Rule" considers $\rho_B = 0.999$ and $v_{\Delta e} = 10$. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^*_{t+1} - R_{t+1}\right]$ measures the relative cost of borrowing in foreign currency from the point of view of banks.

FIGURE 22. *Commodity price shock*



Note. "Δe Rule" considers $\rho_B = 0.999$ and $\upsilon_{\Delta e} = 10$ and "Spread Rule" calibrates $\rho_B = 0.999$ and $\upsilon_{\text{spread}} = 7$. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^*_{t+1} - R_{t+1}\right]$ measures the relative cost of borrowing in foreign currency from the point of view of banks.

FIGURE 23. *Foreign output shock*



Note. "Δe Rule" considers $\rho_B = 0.999$ and $\upsilon_{\Delta e} = 10$ and "Spread Rule" calibrates $\rho_B = 0.999$ and $\upsilon_{\text{spread}} = 7$. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^*_{t+1} - R_{t+1}\right]$ measures the relative cost of borrowing in foreign currency from the point of view of banks.

Appendix C. Model Solution

C.1 The Financial System

Solving Bank's Problem. Recursive version for banker's problem:

$$V_t = \max_{l_t, l_t^*, b_t, d_t, d_t^*} \mathbb{E}_t\left[\Lambda_{t,t+1}\{(1-\sigma)n_{t+1} + \sigma V_{t+1}\}\right]$$

subject to:

$$l_t + e_t l_t^* + b_t = n_t + d_t + e_t d_t^*$$
$$n_{t+1} = R_{t+1}^l l_t + R_{t+1}^{l*} e_{t+1} l_t^* + R_{t+1}^b b_t - R_{t+1} d_t - e_{t+1} R_{t+1}^* d_t^*$$
$$x_t = \frac{e_t d_t^* - e_t l_t^*}{l_t + e_t l_t^* + b_t}$$
$$V_t \geq \Theta(x_t)[l_t + \varpi^* e_t l_t^* + \varpi^b b_t]$$

Let $\psi_t = \frac{V_t}{n_t}$, $\phi_t^l = \frac{l_t}{n_t}$, $\phi_t^l = \frac{e_t l_t^*}{n_t}$, and $\phi_t^b = \frac{b_t}{n_t}$, then the objective function can be rewritten as

$$\psi_t = \mathbb{E}_t\left[\Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1})\frac{n_{t+1}}{n_t}\right]$$

Using the law of motion for bank's net worth, we can rearrange:

$$\frac{n_{t+1}}{n_t} = R_{t+1}^l \frac{l_t}{n_t} + R_{t+1}^{l*} \frac{e_{t+1}}{e_t}\frac{e_t l_t^*}{n_t} + R_{t+1}^b \frac{b_t}{n_t} - R_{t+1}\frac{d_t}{n_t}$$
$$-R_{t+1}^* \frac{e_{t+1}}{e_t}\frac{(l_t + e_t l_t^* + b_t)}{n_t}\frac{(e_t d_t^* - e_t l_t^* + e_t l_t^*)}{l_t + e_t l_t^* + b_t}$$
$$\frac{n_{t+1}}{n_t} = R_{t+1}^l \phi_t^l + \frac{e_{t+1}}{e_t}(R_{t+1}^{l*} - R_{t+1}^*)\phi_t^{l*} + R_{t+1}^b \phi_t^b - R_{t+1}[\frac{l_t}{n_t} + \frac{e_t l_t^*}{n_t}$$
$$+\frac{b_t}{n_t} - 1 - \frac{e_t d_t^*}{n_t}] - \frac{e_{t+1}}{e_t} R_{t+1}^*[\phi_t^l + \phi_t^{l*} + \phi_t^b]x_t$$
$$\frac{n_{t+1}}{n_t} = R_{t+1}^l \phi_t^l + \frac{e_{t+1}}{e_t}(R_{t+1}^{l*} - R_{t+1}^*)\phi_t^{l*} + R_{t+1}^b \phi_t^b$$
$$-R_{t+1}[\phi_t^l + \phi_t^b - 1 - [\phi_t^l + \phi_t^{l*} + \phi_t^b]x_t] - \frac{e_{t+1}}{e_t} R_{t+1}^*[\phi_t^l + \phi_t^{l*} + \phi_t^b]x_t$$
$$\frac{n_{t+1}}{n_t} = [R_{t+1}^l - R_{t+1}]\phi_t^l + \left[\frac{e_{t+1}}{e_t}\left(R_{t+1}^{l*} - R_{t+1}^*\right)\right]\phi_t^{l*}$$
$$+[R_{t+1}^b - R_{t+1}]\phi_t^b + \left[R_{t+1} - \frac{e_{t+1}}{e_t}R_{t+1}^*\right](\phi_t^l + \phi_t^{l*} + \phi_t^b)x_t + R_{t+1}$$

Thus, bank's problem can be rewritten as the following form:

$$\psi_t = \max_{\phi_t^l, \phi_t^{l*} \phi_t^b, x_t} \mu_t^l \phi_t^l + (\mu_t^{l*} + \mu_t^{d*})\phi_t^{l*} + \mu_t^b \phi_t^b + \mu_t^{d*}(\phi_t^l + \phi_t^{l*} + \phi_t^b)x_t + \upsilon_t$$

subject to:

$$\psi_t - \Theta(x_t)[\phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b] \geq 0$$

Where

$$\mu_t^l = \mathbb{E}_t\left[\Omega_{t+1}\left(R_{t+1}^l - R_{t+1}\right)\right] \quad (1)$$

$$\mu_t^{l*} = \mathbb{E}_t[\Omega_{t+1}(\frac{e_{t+1}}{e_t}R_{t+1}^{l*} - R_{t+1})] \quad (2)$$

$$\mu_t^b = \mathbb{E}_t[\Omega_{t+1}(R_{t+1}^b - R_{t+1})] \quad (3)$$

$$\mu_t^{d*} = \mathbb{E}_t[\Omega_{t+1}(R_{t+1} - \frac{e_{t+1}}{e_t}R_{t+1}^*)] \quad (4)$$

$$v_t = \mathbb{E}_t[\Omega_{t+1}R_{t+1}] \quad (5)$$

$$\Omega_{t+1} = \Lambda_{t,t+1}(1 - \sigma + \sigma\psi_{t+1}) \quad (6)$$

We can interpret Ω_{t+1} as the stochastic discount factor of the banker, μ_t^l as the excess return of domestic currency loans over home deposit, $\mu_t^l$is the excess return of foreign currency loans over home deposit, μ_t^b the excess return of sterilized bonds over home deposit, and μ_t^{d*} as the cost advantage of foreign currency debt over home deposit. Note that at the optimal ratios, the following equation will be satisfied:

$$\psi_t = \mu_t^l\phi_t^l + (\mu_t^{l*} + \mu_t^{d*})\phi_t^{l*} + \mu_t^b\phi_t^b + \mu_t^{d*}(\phi_t^l + \phi_t^{l*} + \phi_t^b)x_t + v_t \quad (7)$$

Let λ_t^b be the Lagrange multiplier of the associated incentive restriction, then the problem becomes:

$$\mathcal{L}_t = max_{\phi_t^l,\phi_t^{l*},\phi_t^b,x_t}\mu_t^l\phi_t^l + (\mu_t^{l*} + \mu_t^{d*})\phi_t^{l*} + \mu_t^b\phi_t^b + \mu_t^{d*}(\phi_t^l + \phi_t^{l*} + \phi_t^b)x_t + v_t$$
$$+\lambda_t^b[\mu_t^l\phi_t^l + (\mu_t^{l*} + \mu_t^{d*})\phi_t^{l*} + \mu_t^b\phi_t^b + \mu_t^{d*}(\phi_t^l + \phi_t^{l*} + \phi_t^b)x_t + v_t$$
$$-\Theta(x_t)(\phi_t^l + \varpi^*\phi_t^{l*} + \varpi^b\phi_t^b)]$$

$$\mathcal{L}_t = max_{\phi_t^l,\phi_t^{l*},\phi_t^b,x_t}(1 + \lambda_t^b)[\mu_t^l\phi_t^l + (\mu_t^{l*} + \mu_t^{d*})\phi_t^{l*} + \mu_t^b\phi_t^b + \mu_t^{d*}(\phi_t^l + \phi_t^{l*} + \phi_t^b)x_t + v_t]$$
$$-\lambda_t^b\Theta(x_t)(\phi_t^l + \varpi^*\phi_t^{l*} + \varpi^b\phi_t^b)$$

Then, the first order conditions (FOCs) for this problem are:

ϕ_t^l: $$(1 + \lambda_t^b)[\mu_t^l + \mu_t^{d*}x_t] - \lambda_t^b\Theta(x_t) = 0$$

ϕ_t^{l*}: $$(1 + \lambda_t^b)[\mu_t^{l*} + \mu_t^{d*} + \mu_t^{d*}x_t] - \varpi^*\lambda_t^b\Theta(x_t) = 0$$

ϕ_t^b: $$(1 + \lambda_t^b)[\mu_t^b + \mu_t^{d*}x_t] - \varpi^b\lambda_t^b\Theta(x_t) = 0$$

x_t: $$(1 + \lambda_t^b)\mu_t^{d*}(\phi_t^l + \phi_t^{l*} + \phi_t^b) - \lambda_t^b(\phi_t^l + \varpi^*\phi_t^{l*} + \varpi^b\phi_t^b)\partial_x\Theta(x_t) = 0$$

slackness: $$\lambda_t^b[\psi_t - \Theta(x_t)(\phi_t^l + \varpi^*\phi_t^{l*} + \varpi^b\phi_t^b)] = 0$$

We assume that $\lambda_t^b > 0$ and the incentive constraint is binding. Thus

$$\mu_t^l + \mu_t^{d*} x_t = \frac{\lambda_t^b}{1+\lambda_t^b} \Theta(x_t)$$

$$\mu_t^{l*} + \mu_t^{d*}(1 + x_t) = \frac{\lambda_t^b}{1+\lambda_t^b} \varpi^* \Theta(x_t)$$

$$\mu_t^b + \mu_t^{d*} x_t = \frac{\lambda_t^b}{1+\lambda_t^b} \varpi^b \Theta(x_t)$$

$$\mu_t^{d*}(\phi_t^l + \phi_t^{l*} + \phi_t^b) = \frac{\lambda_t^b}{1+\lambda_t^b} (\phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b) \partial_x \Theta(x_t)$$

$$\Theta(x_t)(\phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b) = (\mu_t^l + \mu_t^{d*} x_t)\phi_t^l + (\mu_t^{l*} + \mu_t^{d*} x_t)\phi_t^{l*} + (\mu_t^b + \mu_t^{d*} x_t)\phi_t^b + \upsilon_t$$

Dividing the first condition by the second and third:

$$\mu_t^{l*} = \varpi^* \mu_t^l - [(1 - \varpi^*)x_t + 1]\mu_t^{d*} \quad (8)$$

$$\mu_t^b = \varpi^b \mu_t^l - (1 - \varpi^b)\mu_t^{d*} x_t \quad (9)$$

Considering the incentive constraint we can rearrange to obtain:

$$\phi_t^l = \Phi_t - \varpi^* \phi_t^{l*} - \varpi^b \phi_t^b \quad (10)$$

$$\Phi_t = \frac{\upsilon_t}{\Theta(x_t) - (\mu_t^l + \mu_t^{d*} x_t)} \quad (11)$$

Note Φ_t defines the maximum weighted leverage ratio induced by the moral hazard problem.[24] We can see that, whenever $\varpi^*, \varpi^b > 0$, private loans and sterilized bonds are substitutes in the portfolio of banks.

[24] Note that this restriction can be rewritten as:

$$l_t \le \theta_t n_t$$

Where $\theta_t = \Phi_t - \varpi^* \phi_t^{l*} - \varpi^b \phi_t^b$. This type of collateral constraint was popularized Kiyotaki and Moore (1997) and is used in Chang (2019) to capture foreign debt limits that are faced by the financial system in emerging economies.

Using the fourth optimality condition:

$$\mu_t^{d*}(\phi_t^l + \phi_t^{l*} + \phi_t^b) = (\mu_t^l + \mu_t^{d*}x_t)\frac{\partial_x \Theta(x_t)}{\Theta(x_t)}\Phi_t$$

$$\mu_t^{d*}(\Phi_t + (1-\varpi^*)\phi_t^{l*} + (1-\varpi^b)\phi_t^b) = (\mu_t^l + \mu_t^{d*}x_t)\frac{\partial_x \Theta(x_t)}{\Theta(x_t)}\Phi_t$$

$$\mu_t^{d*}(\Phi_t + (1-\varpi^*)\phi_t^{l*} + (1-\varpi^b)\phi_t^b) = (\mu_t^l + \mu_t^{d*}x_t)\frac{\partial_x \Theta(x_t)}{\Theta(x_t)}\Phi_t$$

$$\mu_t^{d*}(1-\varpi^*)\phi_t^{l*} + \mu_t^{d*}(1-\varpi^b)\phi_t^b = [(\mu_t^l + \mu_t^{d*}x_t)\frac{\partial_x \Theta(x_t)}{\Theta(x_t)} - \mu_t^{d*}]\Phi_t$$

Hence, the fifth equation for solving bank's problem is:[25]

$$(1-\varpi^*)\phi_t^{l*} + (1-\varpi^b)\phi_t^b = \left[\left(\frac{\mu_t^l}{\mu_t^{d*}} + x_t\right)\frac{\partial_x \Theta(x_t)}{\Theta(x_t)} - 1\right]\Phi_t \qquad (12)$$

Financial System Aggregation. We have solved the problem for an individual bank but not for the aggregate banking sector. From eq. (8), we see that the determination of the foreign debt - weighted asset ratio does not depend on bank-specific factors, then this equation is also satisfied at entire banking sector. The same logic applies for eq. (9), eq. (10), eq. (12). Then,

$$\phi_t^l = \frac{L_t}{N_t} \qquad (13)$$

$$\phi_t^{l*} = \frac{e_t L_t^*}{N_t}\frac{L_t}{N_t} \qquad (14)$$

$$\phi_t^b = \frac{B_t}{N_t}\frac{L_t}{N_t} \qquad (15)$$

$$x_t = \frac{e_t D_t^* - e_t L_t^*}{L_t + e_t L_t^* + B_t}\frac{L_t}{N_t} \qquad (16)$$

Since the aggregate level of sterilized bonds B_t is determined by the monetary authority and N_t is a state variable, then, in the whole financial system, ϕ_t^b is given. However, now the vector $(R_t^l, R_t^{l*}, R_t^b)$is not given anymore. The equations which help in the determination of this vector is

[25] Note that if $1 = \varpi^*$and $1 = \varpi^b$, we arrive at a solution similar to that of Aoki et al. (2018):

$$1 = (\frac{\mu_t^l}{\mu_t^{d*}} + x_t)\frac{\partial_x \Theta(x_t)}{\Theta(x_t)}$$

If $\varpi^* = \varpi^b = 1$, we get the same solution of Aoki et al. (2018) for the whole financial system since returns are the same across different types of assets.

the law of motion of the aggregated bank's net worth and credit demand functions. The aggregate net worth of banks evolves according to

$$N_{t+1} = \sigma(R^l_{t+1}L_t + R^{l*}_{t+1}e_{t+1}L^*_t + R^b_{t+1}B_t - R_{t+1}D_t - e_{t+1}R^*_{t+1}D^*_t)$$
$$+\xi(R^l_{t+1}L_t + R^{l*}_{t+1}e_{t+1}L^*_t + R^b_{t+1}B_t)$$
$$N_{t+1} = (\sigma+\xi)\left(R^l_{t+1}L_t + R^{l*}_{t+1}e_{t+1}L^*_t + R^b_{t+1}B_t\right) - \sigma R_{t+1}D_t$$
$$- \sigma e_{t+1}R^*_{t+1}D^*_t \qquad (17)$$

Aggregate Currency Mismatch - Case 3. Given x_t and n_t,

$$V_t = \max_{l_t,l^*_t,b_t,d_t,d^*_t} \mathbb{E}_t\left[\Lambda_{t,t+1}\{(1-\sigma)n_{t+1} + \sigma V_{t+1}\}\right]$$
subject to:
$$l_t + e_t l^*_t + b_t = n_t + d_t + e_t d^*_t$$
$$n_{t+1} = R^l_{t+1}l_t + R^{l*}_{t+1}e_{t+1}l^*_t + R^b_{t+1}b_t - R_{t+1}d_t - e_{t+1}R^*_{t+1}d^*_t$$
$$V_t \geq \Theta(x_t)[l_t + \varpi^* e_t l^*_t + \varpi^b b_t]$$

Let $\psi_t = \frac{V_t}{n_t}$, $\phi^l_t = \frac{l_t}{n_t}$, $\phi^l_t = \frac{e_t l^*_t}{n_t}$, and $\phi^b_t = \frac{b_t}{n_t}$, then the objective function can be rewritten as

$$\psi_t = \mathbb{E}_t\left[\Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1})\frac{n_{t+1}}{n_t}\right]$$

Moreover, let $\phi^{d*}_t = \frac{e_t d^*_t}{n_t}$

$$\frac{n_{t+1}}{n_t} = R^l_{t+1}\phi^l_t + R^{l*}_{t+1}\frac{e_{t+1}}{e_t}\phi^*_t + R^b_{t+1}\phi^b_t - R_{t+1}\frac{d_t}{n_t} - R^*_{t+1}\frac{e_{t+1}}{e_t}\phi^{d*}_t$$
$$\frac{n_{t+1}}{n_t} = R^l_{t+1}\phi^l_t + R^{l*}_{t+1}\frac{e_{t+1}}{e_t}\phi^*_t + R^b_{t+1}\phi^b_t - R_{t+1}[\phi^l_t + \phi^{l*}_t + \phi^b_t - 1 - \phi^{d*}_t]$$
$$- R^*_{t+1}\frac{e_{t+1}}{e_t}\phi^{d*}_t$$
$$\frac{n_{t+1}}{n_t} = [R^l_{t+1} - R_{t+1}]\phi^l_t + \left[R^{l*}_{t+1}\frac{e_{t+1}}{e_t} - R_{t+1}\right]\phi^*_t + [R^b_{t+1} - R_{t+1}]\phi^b_t$$
$$+\left[R_{t+1} - R^*_{t+1}\frac{e_{t+1}}{e_t}\right]\phi^{d*}_t + R_{t+1}$$

Then, the bank's problem can be rewritten as

$$\psi_t = \max_{\phi^l_t,\phi^{l*}_t\phi^b_t,\phi^{d*}_t} \mu^l_t\phi^l_t + \mu^{l*}_t\phi^{l*}_t + \mu^b_t\phi^b_t + \mu^{d*}_t\phi^{d*}_t + \upsilon_t$$
subject to:
$$\psi_t - \Theta(x_t)[\phi^l_t + \varpi^*\phi^{l*}_t + \varpi^b\phi^b_t] \geq 0$$

FOCs

$$\phi_t^l: \qquad (1+\lambda_t^b)\mu_t^l - \lambda_t^b \Theta(x_t) = 0$$
$$\phi_t^{l*}: \qquad (1+\lambda_t^b)\mu_t^{l*} - \varpi^* \lambda_t^b \Theta(x_t) = 0$$
$$\phi_t^b: \qquad (1+\lambda_t^b)\mu_t^b - \varpi^b \lambda_t^b \Theta(x_t) = 0$$
$$\phi_t^{d*}: \qquad (1+\lambda_t^b)\mu_t^{d*} = 0$$
$$\text{slackness:} \quad \lambda_t^b[\psi_t - \Theta(x_t)(\phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b)] = 0$$

Rearranging

$$\mu_t^{d*} = 0$$
$$\mu_t^{l*} = \varpi^* \mu^l$$
$$\mu_t^b = \varpi^b \mu^l$$

Thus

$$\psi = \mu^l \Phi_t + v_t \tag{18}$$

Hence,

$$\Phi_t = \frac{v_t}{\Theta(x_t) - \mu^l} \tag{19}$$

C.2 Solving Worker's Problem

Objective Function:

$$U_t = (1-\beta)\mathbb{E}_t\left[\sum_{j=0}^{\infty} \beta^j \frac{1}{1-\gamma}\left(C_{t+j} - \mathcal{H}C_{t+j-1} - \frac{\zeta_0}{1+\zeta} H_{t+j}^{1+\zeta}\right)^{1-\gamma}\right]$$

Budget Restriction:

$$C_t + D_t + B_t^g + e_t\left[D_t^{*,h} + \frac{\kappa_{D*}}{2}\left(D_t^{*,h} - \overline{D}^{*,h}\right)^2\right] + \left[\mathcal{S}_t + \frac{\kappa_S}{2}\left(\mathcal{S}_t - \overline{\mathcal{S}}\right)^2\right] + T_t$$
$$= w_t H_t + \Pi_t + R_t D_{t-1} + R_t^* e_t D_{t-1}^{*,h} + R_t^{knc} \mathcal{S}_{t-1}$$

First Order Conditions:

$$\mathbb{E}_t u_{ct} w_t = \zeta_0 H_t^{\zeta}\left(C_t - \mathcal{H}C_{t-1} - \frac{\zeta_0}{1+\zeta} H_t^{1+\zeta}\right)^{-\gamma} \tag{1}$$

$$1 = \mathbb{E}_t[R_{t+1}\Lambda_{t,t+1}] \tag{2}$$

$$D_t^{*,h} = \overline{D}^{*,h} + \frac{\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{e_{t+1}}{e_t} R_{t+1}^* - R_{t+1}\right)\right]}{\kappa_{D*}} \tag{3}$$

$$\mathcal{S}_t = \overline{\mathcal{S}} + \frac{\mathbb{E}_t\left[\Lambda_{t,t+1}(R^S_{t+1} - R_{t+1})\right]}{\kappa_S} \tag{4}$$

with

$$u_{ct} = \left(C_t - \mathcal{H}C_{t-1} - \frac{\zeta_0}{1+\zeta}H_t^{1+\zeta}\right)^{-\gamma} - \mathcal{H}\beta\mathbb{E}_t\left(C_{t+1} - \mathcal{H}C_t - \frac{\zeta_0}{1+\zeta}H_{t+1}^{1+\zeta}\right)^{-\gamma} \tag{5}$$

$$\Lambda_{t,t+1} = \beta\frac{u_{c,t+1}}{u_{ct}} \tag{6}$$

C.3 Price Setting

Given $p^{nc}_{j,t-1}$, $k^{nc}_{j,t-1}$, $\mathcal{S}_{j,t-1}$, and $\mathcal{F}_{j,t-1}$, a representative intermediate good producer chooses $\{h_{j,t+s}$, $m_{j,t+s}$, $p^{nc}_{j,t+s}$, $y^{nc}_{j,t+s}$, $k^{nc}_{j,t+s}$, $\mathcal{S}_{j,t+s}$, $\mathcal{F}_{j,t+s}\}_{s\geq 0}$ to maximize

$$\max\ \mathbb{E}_t\left[\sum_{s=0}^{\infty}\Lambda_{t,t+s}\{\frac{p^{nc}_{j,t+s}}{P^{nc}_{t+s}}y^{nc}_{j,t+s} - w_{t+s}h_{j,t+s} - e_{t+s}m_{j,t+s} - \Theta_{t+s}\left(\frac{p^{nc}_{t+s}}{p^{nc}_{t+s-1}}\right) + q^{nc}_{t+s}\lambda_{nc}k^{nc}_{j,t+s-1} - R^S_{t+s}\mathcal{S}_{j,t+s-1} - R^F_{t+s}\mathcal{F}_{j,t+s-1}\}\right]$$

Subject to:

$$0 = y^{nc}_{j,t} - A^{nc}_t\left(\frac{k^{nc}_{j,t-1}}{\alpha_k}\right)^{\alpha_k}\left(\frac{m_{j,t}}{\alpha_m}\right)^{\alpha_m}\left(\frac{h_{j,t}}{1-\alpha_k-\alpha_m}\right)^{1-\alpha_k-\alpha_m}$$

$$0 = y^{nc}_{j,t} - \left(\frac{p^{nc}_{j,t}}{P^{nc}_t}\right)^{-\eta}Y^{nc}_t$$

$$0 = \mathcal{S}_{j,t} + \mathcal{F}_{j,t} - q^{nc}_t k^{nc}_{j,t}$$

Denoting the Lagrangian multipliers: mc_t, $\mathcal{L}_{1t}$, and $\mathcal{L}_{2t}$respectively, and let define

$$z_t = mc_t A_t(k^{nc}_{jt-1})^{\alpha_k-1}\left(\frac{m_{jt}}{\alpha_m}\right)^{\alpha_m}\left(\frac{h_{jt}}{1-\alpha_k-\alpha_m}\right)^{1-\alpha_k-\alpha_m}$$

$$R^{knc}_t = \frac{\lambda_{nc}q^{nc}_t + z_t}{q^{nc}_{t-1}}$$

The necessary conditions are:

$$h_{j,t}\colon 0 = -w_t + mc_t A^{nc}_t\left(\frac{k^{nc}_{j,t-1}}{\alpha_k}\right)^{\alpha_k}\left(\frac{m_{j,t}}{\alpha_m}\right)^{\alpha_m}h_{j,t}^{-\alpha_k-\alpha_m}$$

$$m_{jt}: 0 = -e_t + mc_t A_t^{nc} m_{j,t}^{\alpha_m - 1} \left(\frac{k_{j,t-1}^{nc}}{\alpha_k}\right)^{\alpha_k} \left(\frac{h_{j,t}}{1-\alpha_k-\alpha_m}\right)^{1-\alpha_k-\alpha_m}$$

$$p_{j,t}^{nc}: 0 = \frac{1}{P_t^{nc}} y_{jt}^{nc} - \frac{1}{p_{j,t-1}^{nc}} \Theta'_t - \mathcal{L}_{1t} \eta (p_{j,t}^{nc})^{-\eta-1} \left(\frac{1}{P_t^{nc}}\right)^{-\eta} Y_t^{nc} + \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{p_{j,t+1}^{nc}}{(p_{j,t}^{nc})^2} \Theta'_{t+1}\right]$$

$$y_{j,t}^{nc}:\ 0 = \frac{p_{j,t}^{nc}}{P_t^{nc}} - mc_t - \mathcal{L}_{1t}$$

$$k_{j,t}^{nc}:\ 0 = -\mathcal{L}_{2t} q_t^{nc} + \mathbb{E}_t \Lambda_{t,t+1} [\lambda_{nc} q_{t+1}^{nc} + z_{t+1}]$$

$$\mathcal{S}_{j,t}:\ 0 = \mathcal{L}_{2,t} - \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^S$$

$$\mathcal{F}_{j,t}:\ 0 = \mathcal{L}_{2,t} - \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^F$$

Along with the three restrictions written above. We can rearrange and aggregate to get the following **optimal conditions**:

$$Y_t^{nc} = A_t^{nc} \left(\frac{K_{t-1}^{nc}}{\alpha_k}\right)^{\alpha_k} \left(\frac{M_t}{\alpha_m}\right)^{\alpha_m} \left(\frac{H_t}{1-\alpha_k-\alpha_m}\right)^{1-\alpha_k-\alpha_m} \quad (7)$$

$$z_t = \alpha_k mc_t \frac{Y_t^{nc}}{K_{t-1}^{nc}} \quad (8)$$

$$e_t = \alpha_m mc_t \frac{Y_t^{nc}}{M_t} \quad (9)$$

$$mc_t = \frac{1}{A_t^{nc}} z_t^{\alpha_k} e_t^{\alpha_m} w_t^{1-\alpha_k-\alpha_m} \quad (10)$$

$$q_t^{nc} K_t^{nc} = \mathcal{S}_t + \mathcal{F}_t \quad (11)$$

$$R_t^{knc} = \frac{\lambda_{nc} q_t^{nc} + z_t}{q_{t-1}^{nc}} \quad (12)$$

$$0 = \mathbb{E}_t [\Lambda_{t,t+1} (R_{t+1}^{knc} - R_{t+1}^S)] \quad (13)$$

$$0 = \mathbb{E}_t [\Lambda_{t,t+1} (R_{t+1}^{knc} - R_{t+1}^F)] \quad (14)$$

Moreover, regarding the optimal pricing

$$\frac{1}{P_t^{nc}}\left(\frac{p_{j,t}^{nc}}{P_t^{nc}}\right)^{-\eta} Y_t^{nc} - \frac{\eta}{p_{j,t}^{nc}}\left(\frac{p_{j,t}^{nc}}{P_t^{nc}} - mc_t\right)\left(\frac{p_{j,t}^{nc}}{P_t^{nc}}\right)^{-\eta} Y_t^{nc}$$
$$- \frac{\kappa}{p_{j,t-1}^{nc}}\left(\frac{p_{j,t}^{nc}}{p_{j,t-1}^{nc}} - 1\right) Y_t^{nc} + \kappa \mathbb{E}_t\left[\Lambda_{t,t+1}\frac{p_{j,t+1}^{nc}}{(p_{j,t}^{nc})^2}\left(\frac{p_{j,t+1}^{nc}}{p_{j,t}^{nc}} - 1\right) Y_{t+1}^{nc}\right] = 0$$

Considering the symmetric equilibrium $p_{j,t}^{nc} = P_t^{nc}$ for all $j \in [0,1]$and denoting $\pi_t = \frac{P_t^{nc}}{P_{t-1}^{nc}} - 1$, then

$$0 = \frac{1}{P_t^{nc}} Y_t^{nc} - \frac{\eta}{P_t^{nc}}(1 - mc_t)Y_t^{nc} - \frac{\kappa}{P_{t-1}^{nc}}\left(\frac{P_t^{nc}}{P_{t-1}^{nc}} - 1\right) Y_t^{nc}$$
$$+\kappa \mathbb{E}_t\left[\Lambda_{t,t+1}\frac{P_{t+1}^{nc}}{(P_t^{nc})^2}\left(\frac{P_{t+1}^{nc}}{P_t^{nc}} - 1\right) Y_{t+1}^{nc}\right]$$
$$0 = Y_t^{nc} - \eta(1 - mc_t)Y_t^{nc} - \kappa(1+\pi_t)\pi_t Y_t^{nc} + \kappa \mathbb{E}_t\left[\Lambda_{t,t+1}(1+\pi_{t+1})\pi_{t+1}Y_{t+1}^{nc}\right]$$
$$0 = 1 - \eta(1 - mc_t) - \kappa(1+\pi_t)\pi_t + \kappa \mathbb{E}_t\left[\Lambda_{t,t+1}(1+\pi_{t+1})\pi_{t+1}\frac{Y_{t+1}^{nc}}{Y_t^{nc}}\right]$$

Hence, we obtain the **Phillips Curve equation**:

$$(1+\pi_t)\pi_t = \frac{1}{\kappa}(1 - \eta + \eta mc_t) + \mathbb{E}_t\left[\Lambda_{t,t+1}(1+\pi_{t+1})\pi_{t+1}\frac{Y_{t+1}^{nc}}{Y_t^{nc}}\right] \tag{15}$$

Intermediate good producers also need to decide the optimal composition for $\mathcal{F}_t$. First note that:

$$R_t^F \mathcal{F}_{j,t-1} = R_t^l l_{j,t-1} + R_t^{l*} e_t l_{j,t-1}^* \tag{16}$$

Then, the composition problem is:

$$\min_{l_{j,t}, l_{j,t}^*} \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^F \mathcal{F}_{j,t} = \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^l l_{j,t} + \mathbb{E}_t \Lambda_{t,t+1} e_{t+1} R_{t+1}^{l*} l_{j,t}^*$$

Subject to:
$\mathcal{F}(l_{j,t}, e_t l_{j,t}^*) \leq \mathcal{F}_{j,t}$

Let $\mathcal{L}_3$ be the Lagrangian multiplier associated with the restriction, then the **optimal conditions** are:

$$0 = \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^l - \mathcal{L}_3 \mathcal{F}_1(l_{j,t}, e_t l_{j,t}^*)$$
$$0 = \mathbb{E}_t \Lambda_{t,t+1} \frac{e_{t+1}}{e_t} R_{t+1}^{l*} - \mathcal{L}_3 \mathcal{F}_2(l_{j,t}, e_t l_{j,t}^*)$$

Since we assume that $\mathcal{F}()$is a homogeneous function, then $\mathcal{L}_3 = \mathbb{E}_t \Lambda_{t,t++1} R_{t+1}^F$ or equivalently

$$\mathcal{F}_1(l_{j,t}, e_t l_{j,t}^*) = \frac{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^l}{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^F}$$

$$\mathcal{F}_2(l_{j,t}, e_t l_{j,t}^*) = \frac{\mathbb{E}_t \Lambda_{t,t+1} \frac{e_{t+1}}{e_t} R_{t+1}^{l*}}{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^F}$$

In our baseline parametrization we use the next CES function

$$\mathcal{F}(l, el^*) = A^e L_t^{1-\delta^f} (e_t L_t^*)^{\delta^f} \tag{17}$$

Hence,

$$l_{j,t} = (1 - \delta^f) \left(\frac{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^F}{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^l} \right) \mathcal{F}_{j,t} \tag{18}$$

$$e_t l_{jt}^* = \delta^f \left(\frac{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^F}{\mathbb{E}_t \Lambda_{t,t+1} \frac{e_{t+1}}{e_t} R_{t+1}^{l*}} \right) \mathcal{F}_{j,t} \tag{19}$$

We finally impose that $\mathcal{S}_t$ is equity so that $R_t^S = R_t^{knc}$.